COMPANION TO TATE'S "FIRST PRINCIPLES OF ARITHMETIC,"

BEING

A TREATISE

ON THE

HIGHER RULES AND OPERATIONS OF ARITHMETIC.

BY

THOMAS TATE,

AUTHOR OF "A SYSTEM OF MENTAL ARITHMETIC;" "THE PHILOSOPHY OF EDUCATION;" "A TREATISE ON THE DIFFERENTIAL AND INTEGRAL CALCULUS;" ETC.

LONDON:
LONGMAN, GREEN, LONGMAN, ROBERTS, AND GREEN.
1863.

ADVERTISEMENT.

The method of teaching Arithmetic from first principles, as given by the Author in his little book on this subject, having been almost universally adopted by the elementary schools of this country, it is believed, that a systematic work, after this method, on the higher rules and operations of Arithmetic, would be acceptable to all really interested in education. In the minutes of the Committee of Council on Education for 1856-7, the Rev. Canon Moseley observes,—"Of the numerous works on education published by Mr. Tate, there are two the influence of which on elementary teaching has been remarkable. The one is his Arithmetic, in which he first, of all the authors who have attempted it in England, succeeded in making Arithmetic the 'Logic of the People' by giving for the rules of Arithmetic such *reasons* as it was possible to teach to poor children; and the other is his 'Exercises in Mechanics,' in which he first taught, under a popular form, how by the common rules of arithmetic, and by geometrical construction with the scale and compasses, calculations in mechanics of great practical value may be made by persons who have no knowledge of mathematics."

CONTENTS.

A TREATISE

ON THE

HIGHER RULES AND OPERATIONS OF ARITHMETIC.

---

1. Definitions and Explanations of Terms and Operations.

1. Addition is the reverse operation of subtraction; thus $5 + 3 = 8$, and $8 - 3 = 5$.

2. We multiply a number when we repeat it a certain number of times; thus 5 multiplied by $3 = 5 \times 3 = 5 + 5 + 5 = 15$. The *multiplier* of a quantity indicates how many times that quantity is taken; thus $14 \times 3 = 3$ times $14 = 42$. Here 14 is called the *multiplicand*, and 42 the *product*. Multiplication is a compendious mode of addition.

3. To divide one quantity by another is to find how many times the one is contained in, or can be taken out of, the other. Thus $15 \div 3 = 5$, means that we have to find the number of times that 3 are contained in 15, or the number of times that 3 must be repeated to make 15. Division, therefore, is the reverse operation of multiplication; thus $5 \times 3 = 15$, and conversely $15 \div 3 = 5$. The number to be divided is called the *dividend*, the number by which it is to be divided is called the *divisor*, and the result is called the *quotient;* thus in the foregoing example, 15 is the dividend, 3 the divisor, and 5 the quotient. Division may also be regarded as a compendious mode of subtraction; thus $15 \div 3$ also means the number of times that 3 can be taken away from 15. As we shall afterwards see, $15 \div 3$ may be written $\frac{15}{3}$.

4. The quotient multiplied by the divisor is equal to the dividend; thus as $5 \times 3 = 15$, so therefore $15 \div 3 = 5$. Division is an indirect operation, for it consists in finding by trial what number multiplied by the divisor will produce the dividend.

5. If a quantity be divided into any given number of equal parts, one of them is said to be a certain part, or fractional part, of the whole quantity. Thus if the quantity be divided into 5 equal parts, one of them is the fifth, or $\frac{1}{5}$, of the quantity; two of them are twice the fifth, or $\frac{2}{5}$, of the quantity; and so on. For example, three times the fifth of 20, or $\frac{3}{5}$ of $20 = 3$ times $4 = 12$.

6. When one quantity exactly contains another, the latter is said to

be an *aliquot* part of the former. Thus 4d. is an aliquot part of 1s., for 4d. is $\frac{1}{3}$ of 12d.; 5s. is an aliquot part of £1, for 5s. is $\frac{1}{4}$ of 20s.; and so on.

When numbers are used in reference to things, such as shillings, or ounces, or feet, they are called *concrete* numbers; but when the figures, 3, 5, 7, &c., are employed, without any such reference, they are called *abstract* numbers. Concrete numbers of the same name or kind of units may be treated as abstract numbers.

7. Two quantities may be compared by finding their difference or how much the one exceeds the other, or by finding their *ratio* or how many times the one is greater or less than the other. Thus taking the quantities 12 and 4; we find their difference to be 8; but their ratio is expressed by saying that the former is 3 times the latter. The ratio of 12 to 9 is expressed by 12 : 9, or by $12 \div 9$; and in this case, 12 are 4 times the 3rd of 9, or $12 = \frac{4}{3}$ of 9. The ratio of £1 to 4s. is expressed by £1 : 4s., or by £1 $\div$ 4s.; and in this case, £1 is 5 times 4s., or £1 = 4s. $\times$ 5. The ratio of 4s. to £1 is expressed by 4s. : 20s., or by 4s. $\div$ 20s., and then 4s. are the fifth of 20s., or 4s. = $\frac{1}{5}$ of 20s. For a full elucidation of the fundamental principles of ratios, the student may consult the Author's System of Mental Arithmetic.

8. If unity, or any thing representing unity, be divided into equal parts, those parts are called fractions. Thus if an apple be divided into four equal parts; one of them is called one-fourth, or $\frac{1}{4}$; two of them are called two-fourths, or $\frac{2}{4}$; three of them three-fours, or $\frac{3}{4}$; and so on. In the fraction $\frac{3}{4}$, the 3 is called the numerator, and the 4 the denominator.

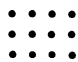
2. Fundamental Axioms.

1. Quantities that are equal to the same thing are equal to one another. If equals be added to equals, the sums are equal. If equals be taken from equals, the remainders are equal. If equals be multiplied or divided by the same number, the results are equal.

2. We may add or subtract quantities in any order; thus $8 + 5 - 3 = 5 - 3 + 8$.

3. Numbers may be multiplied in any order; thus $4 \times 3 = 3 \times 4$.

This is graphically shown by the annexed arrangement of dots. The *whole number* is made up of 4 dots taken 3 times, and at the same time of 3 dots taken 4 times; therefore, 3 times 4 = 4 times 3.



It is obvious that proofs of this kind are not restricted to the particular number or arrangement of the dots employed; so that a proposition, having been proved for one case, will hold true for all other cases.

4. The division of any quantity by a number is the same as taking a certain part of that quantity indicated by the units of the divisor.

Thus $12 \div 4$ is the same as the 4th of 12. The foregoing arrangement of dots shows, that *the number of dots in one vertical row* is the number of times that 4 dots are contained in the whole; and also that *the number of dots in one vertical row* is the 4th of the whole.

Again, any quantity divided by 7, is the same as the 7th of that quantity; for, in matter of fact, *both* operations consist in finding a number which multiplied by 7 shall produce the quantity. For example, the two expressions, 2s. 6d. $\div$ 5, and the 5th of 2s. 6d., lead to the same operation, viz., the finding of a sum of money which multiplied by 5 shall give 2s. 6d.

5. Numbers may be multiplied or divided in any order. Thus to calculate $\frac{8 \times 3}{4}$, we have

Multiplying first, $\frac{8 \times 3}{4} = \frac{24}{4} = 6$; Dividing first, $\frac{8 \times 3}{4} = 2 \times 3 = 6$.

This arrangement of dots shows, that the 4th of 3 times 8 dots = the number in one group, and also that 3 times the 4th of 8 = the number in one group; therefore, the 4th of 3 times 8 = 3 times the 4th of 8.

• •   • •   • •   • •
• •   • •   • •   • •
• •   • •   • •   • •

In order to give another proof of this important principle, let it be required to find the cost of 3 articles, when the cost of 4 articles is 8s.

First, Cost 4 = 8s.
∴ „ 12 = 3 times 8s. = 24s.
∴ „ 3 = $\frac{1}{4}$ of 24s. = 6s.

where the process consists in multiplying 8s. by 3, and then dividing the result by 4.

Second, Cost 4 = 8s.
∴ „ 1 = $\frac{1}{4}$ of 8s. = 2s.
∴ „ 3 = 3 times 2s. = 6s.

where the process consists in dividing 8s. by 4, and then multiplying the result by 3. But both processes lead to the same result, that is, they give the value of the same thing, viz., the cost of 3 articles.

It follows from this axiom, that when the product of two quantities is to be divided by any number, we divide *one* of the factors; thus $(3 \times 8) \div 4 = 3 \times 2$.

6. We may multiply one number by another, by successively multiplying by the factors of the multiplier.

Thus since 8 are 4 times 2, we may find the product of 3 by 8, by first multiplying 3 by 2, and then multiplying this product by 4; that is, $3 \times 8 = 3 \times 2 \times 4$. As shown in the arrangement of 24 dots in the foregoing diagram.

As a further illustration of this principle, let it be required to find the cost of 12 yards of cloth at 3s. 2d. each yard.

First, Cost 1 = 3s. 2d.
∴ „ 12 = 12 times 3s. 2d. = £1 18s.

where we multiply 3s. 2d. by 12.

Second Cost 1 = 3s. 2d.
∴ „ 3 = 3 times 3s. 2d. = 9s. 6d.
∴ „ 12 = 4 times 9s. 6d. = £1 18s.

where we multiply 3s. 2d. successively by 3 and 4, the factors of 12.

It follows from this axiom, that when the product of two quantities is to be multiplied by any number, we multiply this number into *one* of the factors; thus $(3 \times 2) \times 4 = 3 \times 8$.

Whereas, in the case, $(3 + 2) \times 4 = 3 \times 4 + 2 \times 4$, we multiply all the numbers within the vinculum by the multiplier. As shown in the following axiom.

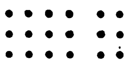
7. Any quantity taken a certain number of times is the same as the parts of that quantity taken the same number of times. Thus $6 \times 3 = (4 + 2) \times 3 = 4 \times 3 + 2 \times 3$.

The succeeding arrangement of dots shows, that 6 times 3 dots make up the same number as 4 times 3 dots added to 2 times 3 dots.

Thus we double a thing when we double all the parts of it; we treble a thing when we treble all the parts of it; and so on.

8. If a quantity be broken up into parts, then the whole quantity will contain the divisor as often as it is contained in the parts; thus

$$18 = 12 + 6, \text{ then } \tfrac{18}{3} = \tfrac{12}{3} + \tfrac{6}{3}.$$



This arrangement of dots shows, that the 3rd of the 18 dots = *the number in one horizontal row*, and also that the 3rd of 12 dots added to the 3rd of 6 dots = *the number in one horizontal row;* therefore the 3rd of 18 dots = the 3rd of 12 dots + the 3rd of 6 dots.

Thus we take the half of a quantity when we take the half of all the parts composing it; we take the third of a quantity when we take the third of all the parts composing it; and so on.

In like manner, $\tfrac{18}{3} = \dfrac{24 - 6}{3} = \tfrac{24}{3} - \tfrac{6}{3}$.

It follows from this axiom, that when the sum or difference of two quantities is to be divided by any number, we divide both quantities by the divisor; thus $\dfrac{12 + 6}{3} = 4 + 2$. Whereas by Ax. 5, $\dfrac{12 \times 6}{3} = 4 \times 6$.

9. We may divide one number by another, by successively dividing by the factors of the divisor. Thus the 8th of 24 may be found by successively dividing 24 by 4 and 2, or $\tfrac{24}{8} = \dfrac{24}{4 \times 2} = \tfrac{6}{2} = 3$.

This arrangement of dots (See Diagram Ax. 5) shows, that the 8th of the 24 dots = *the number in a vertical row*, also that the 4th of the 24 dots is the number in one group, and that the half of this number = *the number in a vertical row.*

As a further illustration of this principle, let it be required to find the cost of one article when the cost of 8 articles is 24s.

First, Cost 8 = 24s.
$\therefore$,, 1 = $\tfrac{1}{8}$ of 24s. = 3s.

where the cost of 1 article is found by dividing 24s. by 8.

Second, Cost 8 = 24s.
$\therefore$,, 2 = $\tfrac{1}{4}$ of 24s. = 6s.
$\therefore$,, 1 = $\tfrac{1}{2}$ of 6s. = 3s.

where the cost of 1 article is found by successively dividing 24s. by 4 and 2, the factors of 8.

10. The division of one number by another may be written in the form of a fraction, having the dividend for the numerator, and the divisor for the denominator. Thus $4 \div 3$ may be written $\frac{4}{3}$.

$$\tfrac{1}{3} \text{ of } 4 = \tfrac{1}{3} \text{ of } (1 + 1 + 1 + 1) = \tfrac{1}{3} + \tfrac{1}{3} + \tfrac{1}{3} + \tfrac{1}{3} = \tfrac{4}{3}$$

$$\text{but by Ax. 4, } 4 \div 3 = \tfrac{1}{3} \text{ of } 4, \ \therefore 4 \div 3 = \tfrac{4}{3}.$$

RULE OF THREE.

(Where a knowledge of Fractions is not required.)

3. In this rule *three* things are given to find a fourth. All questions of this kind may be readily solved by multiplication and division, or by some of the first four rules of arithmetic, without the aid of the technical rule of proportion.

4. BY MULTIPLICATION.

1. If 6 glasses cost 5s. 9d., what will 18 cost?

Cost 6 glasses = 5s. 9d.
∴ „ 18 „ = 3 times 5s. 9d. = 17s. 3d.

Three times the number must cost *three* times the price, that is, 18 must cost 3 times as much as 6.

2. If 7 dishes cost 3s. 8d., what will 35 cost? *Ans.* 18s. 4d.

3. If a man's wages for 3 days be 8s. 6d., what will he earn in 30 days?

Wages for 3 days = 8s. 6d.
∴ „ 30 „ = 10 times 8s. 6d. = £4 5s.

4. If 5 cwt. of sugar cost £9 2s. 6½d., what will 40 cwt. cost? *Ans.* £73 0s. 4d.

5. If a man can build 24 yards of wall in 5 days, how many yards will he build in 30 days?

No. yds in 5 days = 24
∴ „ 30 „ = 6 times 24 = 144.

6. If a man can walk 73 miles in 2 days, what distance will he walk in 6 days? *Ans.* 219 miles.

7. If 2 articles cost 3d., how many can be got for 18d.

No. articles for 3d. = 2
∴ „ 18d. = 6 times 2 = 12.

Here 6 times the money must buy 6 times the number of articles.

8. If 7 lbs. of rice cost 10d., how many lbs. can be got for 3s. 4d. *Ans.* 28.

9. If a man can walk 5 miles in 2 hours, in what time will he walk 45 miles?

No. hours for 5 miles = 2
∴ „ 45 „ = 9 times 2 = 18 hours.

Here 9 times the distance will take 9 times the time.

10. If a man spend £3 in 7 days, how long will he take to spend £36? *Ans.* 84 days

11. If a man can make 4 chairs in 6 days, how many days will he take to make 48 chairs? *Ans.* 72 days.

12. The value of a certain number of articles at 3s. each is 16s. 4d., what will be their value when they are at 15s. each?

Value at 3s. each = 16s. 4d.
∴ „ 15s. „ = 5 times 16s. 4d. = £4 1s. 8d.

Here the price of each article being increased 5 times, the value of the whole must also be increased 5 times.

13. The value of a certain number of articles at 4d. each is £1 3s. 8½d., required their value when they are at 2s. each? *Ans.* £7 2s. 3d.

All the foregoing examples belong to what is called *direct* ratio, that is,

the cost, for example, increases with the number of articles, or is *directly* as the number.

14. If 6 men can complete a certain job in $17\frac{1}{2}$ days, in what time will 1 man do it?

No. days for 6 men $= 17\frac{1}{2}$

$\therefore$,, 1 man $= 6$ times $17\frac{1}{2} = 105$ days.

Here 1 man must take 6 times as long as 6 men.

This is an example belonging to what is called *inverse* or *indirect* ratio, where the time *increases* with the *decrease* of the number of men, or in other words where the time is *inversely* as the number of men.

Without any regard to these technical distinctions, the best guide to the pupil is simply a due attention to the nature of the question.

15. If 15 men can dig a trench in $13\frac{1}{2}$ days, how long will it take 3 men? *Ans.* $67\frac{1}{2}$ days.

16. A man completes a certain journey in 25 hours when he travels at the rate of 24 miles an hour, how long will he take when he only travels at the rate of 4 miles an hour?

No. hours when the rate is 24 miles $= 25$

$\therefore$,, ,, 4 ,, $= 6$ times $25 = 150$ hours.

Here with only one-sixth the speed he must take 6 times as long.

17. If a man spend a certain sum of money in 65 days, when he expends 35s. a day; how long would the money last him if he were to expend only 7s. a day? *Ans.* 325 days.

18. If 1 article cost 3s. 4d., what will 5 articles cost?

Cost 1 article $=$ 3s. 4d.

$\therefore$,, 5 articles $= 5$ times 3s. 4d. $=$ 3s. 4d. $\times 5 =$ 16s. 8d.

The cost of any given number of articles is found by multiplying the cost of 1 by the given number of articles; thus, in this example, the cost of 5 articles is found by multiplying 3s. 4d. by 5.

19. If 1 article cost £2 7s. 2d., what will 36 articles cost? *Ans.* £84 18s.

5. By Division.

1. If 20 articles cost 5s. 3d., what will 5 cost?

Cost 20 $=$ 5s. 3d.

$\therefore$,, 5 $= \frac{1}{4}$ of 5s. 3d $=$ 1s. $3\frac{3}{4}$d.

Here the fourth of the number must cost the fourth of the money, that is, the cost of 5 must be the fourth of the cost of 20.

2. If a dozen bottles of wine cost £1 17s., what should I pay for 4 bottles at the same rate? *Ans.* 12s. 4d.

3. If a man spend £150 10s. 6d. in 52 weeks, how much is that for 13 weeks?

Money spent in 52 weeks $=$ £150 10s. 6d.

$\therefore$,, 13 $= \frac{1}{4}$ of £150 10s. 6d. $=$ £37 12s. $7\frac{1}{2}$d.

4. If a man's wages for 30 days be £3 12s., what will he get for 6 days? *Ans.* 14s. $4\frac{4}{5}$d.

5. If a man can build 45 yards of wall in 20 days, how many yards will he build in 4 days? *Ans.* 9.

6. What will be the price of 2 oz. of tea at 2s. 10d. per lb.?

1 lb. $=$ 16 oz.; then price 16 oz. $=$ 2s. 10d.

$\therefore$,, 2 oz. $= \frac{1}{8}$ of 2s. 10d. $= 4\frac{1}{4}$d.

Here *one eighth* the weight will be *one eighth* the price.

7. If a cwt. of sugar cost £2 12s., how much will 14 lbs. cost?
Ans. 6s. 6d.

8. What will be the price of 4 oz. of tea at 5s. 6d. per lb.?
Ans. 1s. $4\frac{1}{2}$d.

9. If 8 articles cost 18s. 4d., what will be the price of 1 article?

Cost 8 = 18s. 4d.

$\therefore$,, 1 = $\frac{1}{8}$ of 18s. 4d. = $\frac{18s.\ 4d.}{8}$ = 2s. $3\frac{1}{2}$d.

Here, to find the price of 1 article, we divide the given cost by the given number of articles. Thus 18s. 4d. being the given cost of 8 articles, we divide 18s. 4d. by 8 to obtain the price of 1 article.

10. What will be the price of 1 lb. of sugar at £2 2s. per cwt?
Ans. $4\frac{1}{2}$d.

11. If 6 lbs. of tobacco cost £1 12s., what will a quarter of a lb. cost?

Cost 6 lbs. = £1 12s.
,, 1 lb. = $\frac{1}{6}$ of £1 12s. = 5s. 4d.
,, $\frac{1}{4}$,, = $\frac{1}{4}$ of 5s. 4d. = 1s. 4d.

12. If 7 yds. of cloth cost £1 16s. 2d., what will a half yard cost?
Ans. 2s. 7d.

13. If the wages of a workman for 1 week is £1, what will he get for a quarter of a day? *Ans.* 10d.

14. If 42 yds. of calico cost 18s., how many yards can be got for 3s.?

No. yds. for 18s. = 42.
,, 3s. = $\frac{1}{6}$ of 42 = 7 yds.

Here *one-sixth* of the money will only buy *one sixth* the number of yards.

15. If 24 lbs. of sugar cost 4s. 6d., how many lbs. can be got for 1s. 6d.? *Ans.* 8 lbs.

16. If the value of goods at 18s. each be £22 4s., what will be their value at 6s. each?

Value at 18s. = £22 4s.
$\therefore$,, 6s. = $\frac{1}{3}$ £22 4s. = £7 8s.

Here the goods at *one-third* the price must be *one-third* the value.

17. If the value of a certain number of articles at 10s. each be £36 13s. 4d., what will be their value at 2s. 6d. each? *Ans.* £9 3s. 4d.

18. If a man can run 20 miles in 2 hours, what time will he take to run 1 mile?

Time for 20 miles = 2 hours = 120 min.
$\therefore$,, 1 ,, = $\frac{1}{20}$ of 120 min. = 6 min.

19. If a man can walk 28 miles in 8 hours, how long will he take to walk 7 miles? *Ans.* 2 hours.

20. If 12 men can do a certain piece of work in 5 days, how many men will do the same in 15 days?

No. men to do it in 5 days = 12
$\therefore$,, ,, 15 ,, = $\frac{1}{3}$ of 12 = 4.

Here as the men have got three times the time to do the work in, it will only take one-third the number of men.

21. If 8 men can frame a roof in 5 days, how many men will it take to frame it in 20 days? *Ans.* 2 men.

6. By Multiplication and Division.

1. If 5 sheep cost £3, what will 7 sheep cost?

Cost 5 sheep = £3

∴ „ 1 „ = $\frac{1}{5}$ of £3 = £$\frac{3}{5}$ = 12s.

∴ „ 7 „ = 7 times 12s. = 12s. × 7 = 84s. = £4 4s.

Here we divide £3 by 5, and then multiply by 7.

Or thus: Cost 5 = £3

∴ „ 35 = 7 times £3 = £3 × 7 = £21

∴ „ 7 = $\frac{1}{5}$ of £21 = £$\frac{21}{5}$ = £4 4s.

Here we multiply £3 by 7, and then divide by 5.

The latter method is generally best, for we avoid the fraction sometimes resulting from the division.

In the first method, we first divide by 5, and then multiply by 7; whereas in the second method, we first multiply by 7, and then divide by 5; showing that the operations of multiplication and division may be performed in any order. It will be understood, therefore, that whenever the nature of the investigation requires us to divide first and multiply last, we may (should it be most convenient) multiply first and divide last.

2. If 6 lbs. of coffee cost 7s. 6d., what will 7 lbs. cost? *Ans.* 8s. 9d.
3. If 8 articles cost 2s. 6d., what will 5 cost? *Ans.* 1s. 6$\frac{3}{4}$d.
4. If 2 lbs. 4 oz. of tea cost 15s., what will 1 lb. 7oz. cost?

Here we first reduce the weights to the same denomination, that is to oz.; thus 2 lbs. 4 oz. = 36 oz., and 1 lb. 7 oz. = 23 oz.

Cost 36 oz. = 15s.

∴ „ 1 „ = $\frac{15}{36}$s.

∴ „ 23 „ = $\frac{15\text{s.} \times 23}{36}$ = 9s. 7d.

Where we multiply 15s. by 23 and divide by 36.

The arithmetical expression for the cost of the 23 oz. may be at once written down as follows: First writing down $\frac{15}{36}$s., which is the cost of 1 oz., then $\frac{15\text{s.} \times 23}{36}$, which is the cost for 23. In this manner, the expression required may be at once written down without giving the steps of the process of reasoning. The following rule may not be without some use.

Rule.—When the value of one quantity is given to find the value of another quantity of the same kind. Bring the two quantities (if necessary) to the same denomination; find an expression for the value of 1, and multiply by the number whose value is required; but as the order of the operations may be changed, it is generally best to perform the multiplication first and then to divide.

5. What will be the cost of 63 lbs. at £3 0s. 8d. per cent.? *Ans.* £1 14s. 1$\frac{1}{2}$d.

6. If 6 articles cost £19, what will 9 cost? *Ans.* £28 10s

Here we find in the usual manner. Cost 9 $= \frac{£19 \times 9}{6}$

But a more simple expression may be found as follows:

$$\text{Cost } 6 = £19$$
$$\therefore \quad ,, \; 18 = 3 \text{ times } £19 = £19 \times 3$$
$$\therefore \quad ,, \; 9 = \tfrac{1}{2} \text{ of } £19 \times 3 = \frac{£19 \times 3}{2}$$

Now the latter expression may be got from the former by *cancelling;* thus

$$\text{Cost } 9 = \frac{£19 \times \overset{3}{\cancel{9}}}{\underset{2}{\cancel{6}}} = \frac{£19 \times 3}{2}$$

where the factors 9 and 6 are each divided by 3.

Hence calculations may be often simplified by cancelling.

7. If 15 cost 7s. 6d., what will 25 cost? *Ans.* 12s. 6d.

8. If a cwt. of sugar cost £3 3s., what will 24 lbs. cost. *Ans.* 13s. 6d.

9. How many oranges can I get for 15s., when 26 oranges cost 2s.?

$$\text{No. oranges for } 2s. = 26$$
$$\therefore \quad ,, \quad ,, \quad 1s. = \tfrac{1}{2} \text{ of } 26 = 13$$
$$\therefore \quad ,, \quad ,, \quad 15s. = 15 \text{ times } 13 = 195$$

where we first divide 26 by 2, and then multiply by 15.

Or thus, No. oranges for 2s. = 26

$$\therefore \quad ,, \quad ,, \quad 30s. = 15 \text{ times } 26 = 390$$
$$\therefore \quad ,, \quad ,, \quad 15s. = \tfrac{1}{2} \text{ of } 390 = 195$$

where we first multiply 26 by 15, and then divide by 2. See remarks to Ex. 1.

The arithmetical expression for the number of oranges required may be at once written down as follows: First writing down $\frac{26}{2}$, which is the No. for 1s., then $\frac{26 \times 15}{2}$, which is the No. for 15s. Hence we have the following rule,

Rule.—The price of a certain number of things being given to find the number which a given sum will purchase. Bring the two given sums of money (if necessary) to the same denomination; find the number which a unit of price will purchase, and multiply by the units in the proposed price.

The following rule is applicable to all questions which can be given under Rule of Three or Proportion.

General Rule.—Bring the quantities to be compared (if necessary) to the same name or denomination; find the value of a unit, and then from this find an expression for the value of the units required.

The rules that are here given are not intended to save the pupil the trouble of reasoning, as most rules unfortunately do, but rather to methodize the process of reasoning which should be adopted in the solution of each problem.

10. How many articles can be got for £6, when 8 articles cost £4? *Ans.* 12.

11. If 4 yds. of cloth cost 6s., how many yards can be got for 9s.
Ans. 6.

12. If a lb., or 16 oz., of tea cost 4s., what weight of this tea can I get for 7s.? *Ans.* 1 lb. 12 oz.

13. If a cwt. of sugar cost £1 12s. 8d., what weight of it can I purchase for 15s. 2d.?

Here £1 12s. 8d. = 392d.; 15s. 2d. = 182d.; and 1 cwt. = 112 lbs.

$$\begin{aligned} &\text{No. lbs. for 392d.} = 112 \\ \therefore \quad &\quad ,, \quad \text{182d.} = \frac{112 \times 182}{392} \text{ (By cancelling)} \\ &\qquad\qquad = \frac{2 \times 182}{7} = 56 \text{ lbs.} \end{aligned}$$

14. How much butter at £2 10s. 4d. per cwt. may be purchased for £1 11s. $5\frac{1}{2}$d.? *Ans.* 70 lbs.

15. If 2 cwt. 1 qr. 3 lbs. of sugar cost £4 5s., what will 1 cwt. 3 qrs. 9 lbs. cost?

Here 2 cwt. 1 qr. 3 lbs. = 255 lbs., and 1 cwt. 3 qrs. 9 lbs. = 205; then

$$\text{Cost 205 lbs.} = \frac{\text{£4 5s.} \times 205}{255} = \frac{\text{£4 5s.} \times 41}{51} = \text{£3 8s. 4d.}$$

In performing the calculation we may either multiply the money by 41, as in compound multiplication, and divide the result by 51, as in compound long division, or we may bring the money to its lowest denomination and then proceed to multiply and divide. Thus

£ s. d.	£ s.
4 5 0	4 5
41	20
51)174 5 0(£3 8s. 4d.	85
153	41
21	85
20	340
425	51)3485(20) 68s. 4d.
408	306 £3 8s. 4d.
17	425
&c.	&c.

16. If 5 lbs. 8 oz. of tea cost £5 10s., what will 7 lbs. 12 oz. cost
Ans. £7 15s.

17. If a man can walk 40 miles in 15 hours, how many miles will he walk in 9 hours?

$$\begin{aligned} &\text{No. of miles in 15 hours} = 40 \\ \therefore \quad &\quad ,, \qquad 3 \quad ,, \quad = \tfrac{40}{5} = 8 \\ \therefore \quad &\quad ,, \qquad 9 \quad ,, \quad = 8 \times 3 = 24. \end{aligned}$$

18. If a man can build 21 yards of walling in 9 days, how many yards will he build in 24 days? *Ans.* 56.

19. If a man can make a dozen chairs in 8 days, how long will he take to make 33?

$$\begin{array}{llll} & \text{No. days for 12 chairs} & = 8 \\ \therefore & \quad ,, \qquad 3 \quad ,, & = \frac{8}{4} = 2 \\ \therefore & \quad ,, \qquad 33 \quad ,, & = 2 \times 11 = 22. \end{array}$$

20. If a man can walk 35 miles in 14 hours, how long will he take to walk 15 miles? *Ans.* 6 hours.

21. If the coach fare for 63 miles be 14s. 7d., what will it be for 27 miles? *Ans.* 6s. 3d.

22. If the railway fare for 24 miles be 3s., what distance can I go for 2s. 3d.?

Here 3s. = 36d.; and 2s. 3d. = 27d.; then

$$\begin{array}{llll} & \text{Distance for } 36d. & = 24 \text{ miles} \\ \therefore & \quad ,, \qquad 9d. & = \frac{1}{4} \text{ of } 24 \text{ miles} = 6 \text{ miles} \\ \therefore & \quad ,, \qquad 27d. & = 3 \text{ times } 6 \text{ miles} = 18 \text{ miles.} \end{array}$$

Or we may at once write down, Dist. for 27d. $= \frac{24 \times 27}{36} = \frac{24 \times 3}{4}$ $= 18$.

23. What will be the rate of charge per mile in the last example? *Ans.* $1\frac{1}{2}$d.

24. If the coach fare for 30 miles be 4s., what distance can I go for 3s. 4d.? *Ans.* 25 miles.

25. If 10 men can do a certain piece of work in 12 days, how long will it take 15 men to do the same?

$$\begin{array}{llll} & \text{No. days for 10 men} & = 12 \\ \therefore & \quad ,, \qquad 5 \quad ,, & = 2 \text{ times } 12 = 24 \\ \therefore & \quad ,, \qquad 15 \quad ,, & = \frac{1}{3} \text{ of } 24 = 8 \text{ days.} \end{array}$$

Here half the men must take twice the time, and thrice the men must take one-third the time. This is a question of inverse ratio or proportion.

26. If 4 men can frame a roof in 6 days, how long will it take 3 men to do it? *Ans.* 8 days.

27. If 8 men can finish a piece of work in 6 days, how many men will it take to do the same in 16 days?

$$\begin{array}{llll} & \text{No. of men for 6 days} & = 8 \\ \therefore & \quad ,, \qquad 1 \text{ day} & = 6 \text{ times } 8 = 8 \times 6 \\ \therefore & \quad ,, \qquad 16 \text{ days} & = \frac{1}{16} \text{ of } 8 \times 6 = \frac{8 \times 6}{16} = 3. \end{array}$$

Here to do the work in 1 day must take 6 times the number of men, &c.

28. If 15 men can dig a trench in 24 days how many men will it take to do the same in 20 days? *Ans.* 18.

29. If the loan of £100 for a year brings an interest of £3, what will be the interest on £15?

$$\begin{array}{llll} & \text{Interest on } £100 & = £3 \\ \therefore & \quad ,, \qquad £5 & = £\frac{3}{20} = 3s. \\ \therefore & \quad ,, \qquad £15 & = 3 \text{ times } 3s. = 9s. \end{array}$$

30. What will be the interest on £60 at £2 for £100, or at 2 per cent.?

$$\begin{array}{llll} & \text{Interest on } £1 & = £\frac{2}{100} \\ \therefore & \quad ,, \qquad £60 & = \frac{£2 \times 60}{100} = £1\ 4s. \end{array}$$

Here the *principal*, £60, is multiplied by the rate per cent., £2, and the result is divided by 100. This division may be performed by the rule of compound long division.

31. What will be the interest on £35 at 4 per cent.? *Ans.* £1 8s.
32. What will be the interest on £45 at 3 per cent.? *Ans.* £1 7s.
33. What will be the interest on £122 at 5 per cent.?

Interest on £100 = £5

∴ „ £1 = £$\frac{5}{100}$ = 1s.

∴ „ £122 = 1s. × 122 = 122s. = £6 2s.

It is useful to remember that the interest on £1 at 5 per cent. is 1s.

34. Required the interest on £173, £342, £460, £671, at 5 per cent.?
Ans. £8 13s., £17 2s., £23, £33 11s.
35. Required the interest on £55, £80, £95, £125, at 4 per cent.?
Ans. £2 4s., £3 4s., £3 16s., £5.

These methods of solution from first principles, besides cultivating the reasoning powers of the pupil, are really the most practical methods, as will be more fully shown in the following examples.

7. *Different methods of solution, whereby the calculation is simplified.*

The following methods of solution *always* lead us to the most expeditious form of calculation.

1. If 8 cwts. of sugar cost £16 9s. 4d., what will 41 cwts. cost?

	£	s.	d.		£	s.	d.
Cost 40 cwts. = 5 times	16	9	4	=	82	6	8
„ 1 „ = $\frac{1}{8}$ of	16	9	4	=	2	1	2
∴ Cost 41 cwts. =					84	7	10

By the usual mode of calculation we should have to multiply by 41 and then to divide by 8, but by the foregoing method we have only to multiply by 5 and then to add the price of 1.

2. If 6 cost 19s. 6d., what will 37 cost? *Ans.* £6 0s. 3d.
3. If 6 cost 16s. 6d., what will 28 cost?

	s.	d.		£	s.	d.
Cost 30 = 5 times	16	6	=	4	2	6
„ 2 = $\frac{1}{3}$ of	16	6	=		5	6
∴ Cost 28 =				3	17	0

Here we find the cost of 30 and then subtract the cost of 2.

4. If 5 cost 2s. 3½d., what will 24 cost? *Ans.* 11s.
5. If a dozen articles cost 13s. 3d,, what will 74 cost? *Ans.* £4 1s. 8½d.
6. If 1 lb. 4 oz. of tea cost 13s. 4d., what will 1 lb. 9 oz. cost?

1 lb. 4 oz. = 20 oz., 1 lb. 9 oz. = 25 oz.

	s.	d.
Cost 20 oz. =	13	4
„ 5 „ = $\frac{1}{4}$ of 13s. 4d. =	3	4
∴ Cost 25 oz. =	16	8

7. If 1 lb. 14 oz. of tea cost £11 6s. 3d., what will 2 lbs. 4 oz. cost? *Ans.* £13 11s. 6d

8. If 49 cost £13 9s. 6d., what will 50 cost? *Ans.* £13 15s.

9. If 10 cost £1 7s. 11d., what will 9 cost? *Ans.* £1 5s. $1\frac{1}{2}$d.

10. If 16 cost £3 8s. $3\frac{1}{2}$d., what will 15 cost? *Ans.* £3 4s.

11. Required the interest on £14 5s. at 5 per cent?

Interest on £1 = 1s.; Ex. 33, Art. 6.

∴ „ £14 = . 14s 0d.

„ 5s. = $\frac{1}{4}$ of 1s. = 3d.

∴ Interest on £14 5s. = 14s. 3d.

12. Required the interest on £15 4s., £42 10s., £67 3s. 4d., at 5 per cent.? *Ans.* 15s. $2\frac{1}{4}$d., £2 2s. 6d., £3 7s. 2d.

VULGAR FRACTIONS.

8. *Formation of Fractions.* A fraction is formed by dividing unity, or any object representing unity, into a certain number of equal parts. Thus if an apple be divided into 5 equal parts, one of those parts will be called one-fifth ($\frac{1}{5}$), two of them will be called two-fiths ($\frac{2}{5}$), three of them three-fifths ($\frac{3}{5}$), and so on. In the fraction $\frac{3}{5}$, the 5 is called the denominator, and the 3 is called the numerator; the denominator, therefore, indicates the number of parts into which the unit is divided, and the numerator indicates how many of those parts are taken.

We add fractions having the same denominator by adding the numerators together, and then putting the common denominator beneath that sum. Thus 3 pieces of an apple, added to 2 pieces of the same size, will give 5 of those pieces, whatever part of the apple each piece may be; thus, $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$. And similarly $\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$.

9. A fraction may also be regarded as a quantity resulting from the division of the numerator by the denominator. Thus $\frac{3}{5}$ means three times the fifth of unity, or any object representing unity; but it may also be taken to mean the fifth of 3 units, that is, three-fifths = the fifth of three.

For example, the three-fifths of a pound is the same as the fifth of three pounds; thus

$$\tfrac{3}{5} \text{ of } £1 = 3 \text{ times } \tfrac{1}{5} \text{ of } 20s. = 12s.,$$
$$\text{and } \tfrac{1}{5} \text{ of } £3 = \tfrac{1}{5} \text{ of } 60s. = 12s.$$

This important principle may be proved in the following manner,

$$\tfrac{1}{5} \text{ of } 3 = \tfrac{1}{5} \text{ of } (1 + 1 + 1) = \tfrac{1}{5} + \tfrac{1}{5} + \tfrac{1}{5} = \tfrac{3}{5}.$$

Hence it follows, that $3 \div 5$, or $\frac{1}{5}$ of 3, may be written $\frac{3}{5}$.

The value of $\frac{3}{5}$ of 3s. 4d. may be calculated in two ways, according to the meaning which we assign to the operation $\frac{3}{5}$, that is to say, whether we take it as 3 times the 5th of 3s. 4d., or as the 5th of 3 times 3s. 4d.

10. *To reduce an improper fraction to a mixed number and conversely.*

An *improper* fraction has its numerator greater than its denominator, as $\frac{8}{3}$.

A *mixed* number is composed of a whole number and a fraction, as $4\frac{2}{3}$.

Rule.—To reduce an improper fraction to a whole or mixed number. Divide the numerator by the denominator for the whole number, and the remainder, if any, placed over the given denominator will be the fractional part.

Thus, $\frac{14}{5} = 2\frac{4}{5}$, here 5 is contained in 14 twice, with the remainder 4, which is placed over the given denominator 5, so that the mixed number is 2 and $\frac{4}{5}$.

$$\text{Proof, } \tfrac{14}{5} = \tfrac{5}{5} + \tfrac{5}{5} + \tfrac{4}{5} = 1 + 1 + \tfrac{4}{5} = 2\tfrac{4}{5}.$$

Rule.—To reduce a mixed number to an improper fraction. Multiply the whole number by the denominator of the fractional part; add the result to the numerator for the new numerator, and retain the denominator.

Thus, $2\frac{3}{5} = \frac{13}{5}$; here 2 multiplied by 5 gives 10, which added to 3 gives 13 for the new numerator, the denominator, 5, being retained.

$$\text{Proof, } 2\tfrac{3}{5} = 1 + 1 + \tfrac{3}{5} = \tfrac{5}{5} + \tfrac{5}{5} + \tfrac{3}{5} = \tfrac{13}{5}.$$

To express a whole number as a fraction with any given denominator; multiply the number by the given denominator and it will give the numerator of the fraction required.

For example, let it be required to express 4 as a fraction with the denominator 6; then $4 = \dfrac{4 \times 6}{6} = \frac{24}{6}$.

Here each whole contains 6 sixths, therefore 4 wholes will contain 4 times 6 sixths or 24 sixths.

Exercises.

1. Reduce to improper fractions; $3\frac{7}{8}$; $5\frac{3}{7}$; $2\frac{3}{10}$; $8\frac{1}{3}$; $21\frac{2}{3}$; $18\frac{2}{5}$; $14\frac{3}{7}$; $16\frac{3}{5}$; $19\frac{3}{8}$; $2\frac{3}{22}$; $1\frac{9}{17}$.

2. Reduce to mixed numbers; $\frac{9}{8}$; $\frac{17}{6}$; $\frac{53}{5}$; $\frac{18}{7}$; $\frac{41}{3}$; $\frac{11}{4}$; $\frac{67}{9}$; $\frac{123}{7}$; $\frac{237}{22}$; $\frac{131}{32}$; $\frac{203}{4}$.

Ans. (1.) $\frac{31}{8}$; $\frac{38}{7}$; $\frac{23}{10}$; $\frac{25}{3}$; $\frac{65}{3}$; $\frac{92}{5}$; $\frac{101}{7}$; $\frac{83}{5}$; $\frac{155}{8}$; $\frac{47}{22}$; $\frac{26}{17}$. (2.) $1\frac{1}{8}$; $2\frac{5}{6}$; $10\frac{3}{5}$; $2\frac{4}{7}$; $13\frac{2}{3}$; $2\frac{3}{4}$; $7\frac{4}{9}$; $17\frac{4}{7}$; $10\frac{17}{22}$; $4\frac{3}{32}$; $50\frac{3}{4}$.

11. *To perform division when there is a remainder.*

After obtaining the integer portion of the quotient, we put the remainder as the numerator and the divisor as the denominator for the fractional portion of the quotient. Thus, $44 \div 7 = \frac{44}{7} = 6\frac{2}{7}$.

12. *To multiply or divide a fraction by a whole number or integer.*

$$\tfrac{2}{5} \times 3 = 3 \text{ times } \tfrac{2}{5} = \tfrac{2}{5} + \tfrac{2}{5} + \tfrac{2}{5} = \tfrac{6}{5}.$$

Here we *multiply the numerator by the integer, and retain the denominator.*

$$\text{Conversely, } \tfrac{6}{5} \div 3 = \tfrac{1}{3} \text{ of } \tfrac{6}{5} = \tfrac{2}{5}.$$

Here we *divide the numerator by the integer, and retain the denominator.*

When the numerator is not divisible by the integer, we multiply the denominator by it; thus $\frac{2}{5} \div 3 = \frac{2}{15}$. See Art. 21.

Ex. $\frac{3}{8} \times 5 = \frac{15}{8} = 1\frac{7}{8}$; $\frac{6}{5} \div 2 = \frac{3}{5}$; $\frac{16}{3} \div 4 = \frac{4}{3} = 1\frac{1}{3}$; $\frac{2}{3} \div 3 = \frac{2}{9}$.

13. *To change the denominator of a fraction.*

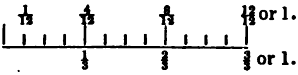
If the numerator and denominator of a fraction be at the same time multiplied or divided by the same number, the value of the fraction will not be altered. Thus $\frac{2}{3} = \frac{8}{12}$; where the numerator, 2, is multiplied

by 4, and the denominator, 3, is also multiplied by 4 to obtain $\frac{8}{12}$; and conversely $\frac{8}{12} = \frac{2}{3}$, where the denominator, 8, is divided by 4, and the denominator, 12, is also divided by 4 to obtain $\frac{2}{3}$.

Proof. $\frac{3}{3} = \frac{12}{12}$; dividing each by 3, Art. 12, we get
$\frac{1}{3} = \frac{1}{3}$ of $\frac{12}{12} = \frac{4}{12}$; multiplying each by 2, Art. 12.
$\frac{2}{3} = \frac{4}{12} \times 2 = \frac{8}{12}$.

The following graphic mode of proof is certainly best adapted for primary instruction.

In this diagram a line, representing unity, is divided into 3 equal parts, forming thirds, and each third is divided into 4 equal parts, forming twelfths. It is at once seen, that two-thirds are equal to eight-twelfths. For other proofs of this kind, see the Author's Principles of Arithmetic, or his " System of Mental Arithmetic."



By multiplying *both* numerator and denominator of the fraction $\frac{2}{3}$, by 2, 3, 4, &c., we have, $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{24} =$ &c. In like manner $\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} =$ &c.

In order to add fractions together, or to subtract one fraction from another, it is necessary that they should be brought to the same denominator, or to a common denominator. Thus $\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$, where the numerator and denominator of the 1st fraction are *both* multiplied by 4, and that of the second fraction by 3; again $\frac{4}{5} - \frac{2}{3} - \frac{12}{15} - \frac{10}{15} = \frac{2}{15}$.

14. *To reduce fractions to their least terms.*

A fraction is brought to its least terms by dividing numerator and denominator by the greatest number that will divide them both.

Thus $\frac{28}{35} = \frac{4}{5}$. Where both terms of the fraction are divided by 7, which in this case is the greatest number that will divide 28 and 35 without a remainder. The 7 is called the greatest common measure (G. C. M.) of the two numbers 28 and 35. In most cases the G. C. M. may be found by simple inspection, or by a few trials. The following observations, however, will be found useful in reducing fractions to their least terms.

1. Any number ending with 5 or 0 is divisible by 5.

Thus 45 and 30 are each divisible by 5.

2. Any number ending with 2, or any other even number, is divisible by 2.

Thus 44 and 62 are each divisible by 2.

3. If the two last figures of a number be divisible by 4, then the number will be divisible by 4. Thus 1316 and 2128 are each divisible by 4.

4. Any number is divisible by 3 or 9 when the sum of its digits is so.

Thus 312 and 63 are each divisible by 3; and 7452 and 81 are each divisible by 9.

5. When the sum of the alternate digits of a number (containing not more than 4 digits) are equal, the number is divisible by 11.

Thus 2761 and 242 are each divisible by 11.

By successive division, therefore, a fraction may be often readily reduced to its least terms.

For example, let it be required to reduce $\frac{3465}{3960}$ to its least terms.

Here by successively dividing by 11, 5, and 9, we have,

$$\frac{3465}{3960} = \frac{315}{360} = \frac{63}{72} = \frac{7}{8}.$$

Exercises. Reduce the following fractions to their least terms, $\frac{176}{286}$, $\frac{90}{108}$, $\frac{220}{572}$, $\frac{1287}{1683}$, $\frac{405}{675}$, $\frac{90}{135}$. *Ans.* $\frac{8}{13}$, $\frac{5}{6}$, $\frac{5}{13}$, $\frac{13}{17}$, $\frac{3}{5}$, $\frac{2}{3}$.

The following method, although often tedious in practice, invariably gives the greatest common measure of the two terms of a fraction.

15. *To find the greatest common measure of the numerator and denominator of a fraction, and thereby to reduce it to its least terms.*

Rule.—Divide the greater term by the less, and the last divisor by the remainder continuously until nothing remains; then the last divisor will be the greatest common measure.

Thus to reduce the fraction $\frac{861}{1107}$ to its least terms, we have

```
861 ) 1107 ( 1
       861
       ———
       246 ) 861 ( 3
             738
             ———
             123 ) 246 ( 2
                   246
                   ——
```

Here the greatest common measure is 123, then

$123\,)\,\frac{861}{1107} = \frac{7}{9}$, which is the fraction in its least terms.

Proof.—Since 123 is the measure of the last dividend, 246, it must also be a measure of, 861, the preceding dividend, for $861 = 3 \times 246 + 123$; but 861 is one of the terms of the fraction. Again, because 123 measures 246 and 861 it must also measure their sum or 1107 the other term of the fraction. And so on similarly to any other case. Moreover 123 is the *greatest* common measure. For, if possible, let us suppose that 124 measures 861 and 1107, then it must also measure their difference 246, and also 3 times 246 or 738; but as it is supposed to measure 861 it must also measure 861 — 738 or 123, which is absurd.

Exercises. Reduce the following fractions to their least terms, $\frac{126}{210}$, $\frac{165}{363}$, $\frac{114}{126}$, $\frac{352}{2000}$, $\frac{352}{968}$, $\frac{432}{1008}$, $\frac{615}{4100}$. *Ans.* $\frac{3}{5}$, $\frac{5}{11}$, $\frac{19}{21}$, $\frac{22}{125}$, $\frac{4}{11}$, $\frac{3}{7}$, $\frac{3}{20}$.

16. *To find the least common multiple of two or more numbers.*

One number is said to be a multiple of another when it can be divided by it without a remainder. Thus 15 is a multiple of 3 and 5.

A common multiple of two or more numbers is a number which contains each of them without a remainder. Thus 12 is a multiple of 2, 3, 4, and 6; 36 is also a multiple of these numbers; but 12 is the least common multiple of them.

Rule.—Arrange the numbers in succession and strike out such of them as are exactly contained in some one of the other numbers. Divide the greatest number of them by some number that will divide them without a remainder. Place the quotients and the undivided numbers under the former, and again strike out those that are exactly contained in some one of them. Proceed in this manner until no number

greater than unity will measure any two of the numbers last found. Then multiply all the numbers last found, and all the divisors continually together, and the result will generally be the least common multiple required.

Ex. Required the least common multiple of 4, 9, 15, 16, 18, 20, 8.

$$\begin{array}{r|l} 5 & 15,\ 16,\ 18,\ 20 \\ \hline 2 & 3,\ 16,\ 18,\ \cancel{4} \\ \hline & \cancel{3},\ 8,\ 9 \end{array}$$

Here we first strike out 4, 9, and 8; because 4 is exactly contained in 20; 9 in 18; and 8 in 16; then the remaining numbers are written down.

Then 5 divides 15 and 20, and in the second line of figures 4 is struck out, because it is exactly contained in 16. Now 2 divides 16 and 18, and in the last line 3 is struck out, because it is exactly contained in 9; then

The least common multiple $= 8 \times 9 \times 2 \times 5 = 720$.

Exercises. Required the least common multiples of the following numbers.

(1.)	12, 20, 30, 36.	*Ans.* 360.	(4.)	8, 12, 28, 24, 16.	*Ans.* 336.
(2.)	28, 8, 64, 7.	,, 448.	(5.)	4, 6, 8, 10, 12.	,, 120.
(3.)	5, 18, 40, 9.	,, 360.	(6.)	2, 3, 4, 5, 6.	,, 60.

17. *To reduce fractions to a common denominator.*

Rule I.—Multiply each numerator into all the denominators, except its own, for a new numerator; and multiply all the denominators together for the common denominator.

Ex. Reduce $\frac{3}{4}$, $\frac{2}{5}$, $\frac{5}{8}$ to fractions having a common denominator.

Here the common denominator $= 4 \times 5 \times 8 = 160$; then

$$\tfrac{3}{4} = \frac{3 \times 5 \times 8}{4 \times 4 \times 8} = \tfrac{120}{160};\ \tfrac{2}{5} = \frac{2 \times 4 \times 8}{5 \times 4 \times 8} = \tfrac{64}{160};\ \tfrac{5}{8} = \frac{5 \times 4 \times 5}{8 \times 4 \times 5} = \tfrac{100}{160}.$$

It is obvious that the values of the fractions are not altered by this process, for the numerator and denominator of each fraction is multiplied by the same number. It will be observed, that this rule does not always give the resulting fractions in their least common denominator.

Rule II.—To reduce fractions to their least common denominator. Find the least common multiple of all the denominators, and this will be the common denominator; divide it by each of the given denominators and then multiply the several quotients by the respective numerators for the new numerators.

Taking the foregoing example, we find the least common multiple of 4, 5, 8 to be 40, which is the least common denominator; then

$$\tfrac{3}{4} = \frac{3 \times 10}{40} = \tfrac{30}{40};\ \tfrac{2}{5} = \frac{2 \times 8}{40} = \tfrac{16}{40};\ \tfrac{5}{8} = \frac{5 \times 5}{40} = \tfrac{25}{40};$$

where the factors 10, 8, 5, in the new numerators, are obtained by dividing 40 by the denominators 4, 5, 8, respectively.

18. *To add and subtract fractions.*

Rule.—Reduce (if necessary) the fractions to their least common denominator, add or subtract the numerators, as the case may be, and retain the common denominator.

It must be observed, that in order to add or subtract fractions, they may be brought to *any common denominator;* but the calculation is generally most easily performed when they are brought to the *least common denominator.*

Ex. 1. $\frac{5}{8}+\frac{3}{16}+\frac{7}{12}+\frac{2}{3}=\frac{30}{48}+\frac{9}{48}+\frac{28}{48}+\frac{32}{48}=\frac{99}{48}=2\frac{1}{16}$.

Here we find the least common multiple of 8, 16, 12, 3 to be 48, which is the common denominator; whence the new numerators are obtained by Rule II., Art. 12. The sum of the numerators is 99 and the common denominator, 48, is retained, giving $\frac{99}{48}$ for the sum of the proposed fractions, which is then brought to the mixed number $2\frac{1}{16}$.

2. $\frac{5}{6}-\frac{3}{4}+\frac{2}{3}=\frac{10}{12}-\frac{9}{12}+\frac{8}{12}=\frac{9}{12}=\frac{3}{4}$.

3. $\frac{3}{4}+\frac{5}{8}+\frac{2}{3}+\frac{1}{6}-\frac{1}{12}=\dfrac{18+15+16+4-2}{24}=\frac{51}{24}=2\frac{1}{8}$.

Or thus. $\quad$ Sum $=\frac{3}{4}+\frac{5}{8}+\frac{2}{3}+\frac{1}{6}-\frac{1}{12}$;

in order to bring all the fractions to whole numbers, we shall multiply both sides of this equality by 24, then

$$24 \text{ times the sum} = \frac{3\times 24}{4}+\frac{5\times 24}{8}+\frac{2\times 24}{3}+\frac{24}{6}-\frac{24}{12}$$
$$= 18+15+16+4-2 = 51$$
$$\therefore \text{ The sum } = \frac{51}{24} = 2\tfrac{1}{8}, \text{ as before.}$$

When any of the quantities are whole or mixed numbers, it is most convenient to add separately the whole and the fractional parts, and then add the two results.

4. $1\frac{2}{3}+3\frac{3}{5}+2\frac{3}{10}+4=10+\frac{2}{3}+\frac{3}{5}+\frac{3}{10}$
$$=10+\frac{20+18+9}{30}=10+\frac{47}{30}=11\tfrac{17}{30}.$$

5. $2\frac{1}{4}\text{d.}+3\frac{2}{5}\text{d.}+2\frac{1}{10}\text{d.}=7\text{d.}+\dfrac{5+8+2}{20}\text{d.}=7\frac{3}{4}\text{d.}$

6. Add together 2s. $2\frac{3}{8}$d., 3s. $7\frac{5}{6}$d., 4s. $3\frac{2}{3}$d., $9\frac{3}{4}$d.

First adding the fractional parts of pence together, we have

$$\tfrac{3}{8}\text{d.}+\tfrac{5}{6}\text{d.}+\tfrac{2}{3}\text{d.}+\tfrac{3}{4}\text{d.}=\frac{9+20+16+18}{24}\text{d.}=\tfrac{63}{24}\text{d.}=2\tfrac{5}{8}\text{d.};$$

carrying the 2d. to the pence, &c., we find the sum = 10s. $11\frac{5}{8}$d.

7. Add together 3 yds. $2\frac{2}{5}$ ft., 2 yds. $2\frac{1}{3}$ ft., $2\frac{7}{10}$ ft., $1\frac{4}{15}$ ft.

First adding the fractional parts of a foot together, we have

$$\tfrac{2}{5}+\tfrac{1}{3}+\tfrac{7}{10}+\tfrac{4}{15}=\frac{12+10+21+8}{30}=\tfrac{51}{30}=\tfrac{17}{10}=1\tfrac{7}{10}\text{ ft.};$$

carrying the 1 ft. to the feet, &c., we find the sum = 7 yards $2\frac{7}{10}$ ft.

8. $8\frac{8}{9}-2\frac{1}{6}-3\frac{2}{3}=3+\frac{8}{9}-\frac{1}{6}-\frac{2}{3}=3+\dfrac{16-3-12}{18}=3\frac{1}{18}$.

9. $3\frac{1}{3}+\frac{1}{6}-2\frac{3}{4}=1+\frac{1}{3}+\frac{1}{6}-\frac{3}{4}=1+\dfrac{4+2-9}{12}=1-\frac{1}{4}=\frac{3}{4}$.

10. From 6s. $4\frac{1}{3}$d. Take 3s. $8\frac{5}{6}$d. —— 2s. $7\frac{1}{2}$d.

Here as $\frac{5}{6}$ is greater than $\frac{1}{3}$, we borrow 1d. from the 4d., and then $1\frac{1}{3}-\frac{5}{6}=\frac{8}{6}-\frac{5}{6}=\frac{1}{2}$; and so on as in compound subtraction.

Before applying the rule, improper fractions should be reduced to mixed numbers.

11. $\frac{75}{8}+\frac{122}{3}-\frac{13}{6}=9\frac{3}{8}+40\frac{2}{3}-2\frac{1}{6}=47+\frac{3}{8}+\frac{2}{3}-\frac{1}{6}=47\frac{7}{8}$.

Exercises. Find the value of

1. $\frac{7}{8} + \frac{5}{6} + \frac{3}{4} + \frac{3}{2}$; $\frac{2}{9} + \frac{5}{6} + \frac{2}{3} + 4$; $\frac{9}{10} - \frac{2}{5}$.
2. $2\frac{2}{7} + 3\frac{1}{3} - 1\frac{1}{6} - \frac{1}{21}$; $\frac{5}{9} - \frac{1}{4} + 2\frac{5}{6} - 1\frac{1}{3} + 4\frac{1}{2} + \frac{5}{18}$.
3. $5\frac{5}{12} - 2\frac{7}{15}$; $4\frac{5}{8} - 2\frac{2}{9} - 1\frac{7}{12}$; $1\frac{8}{7} - 2\frac{3}{4} + 1\frac{3}{14}$.
4. Add 3s. $2\frac{5}{6}$d., 2s. $7\frac{3}{5}$d., 5s. $9\frac{4}{15}$d., $5\frac{1}{10}$d., $2\frac{1}{3}$d.
5. From 5s. $4\frac{7}{8}$d. take 4s. $9\frac{5}{6}$d.; 1s. $2\frac{2}{3}$d. + $5\frac{5}{6}$d. — $9\frac{1}{4}$d.
6. 1s. $4\frac{3}{10}$d. — $6\frac{3}{4}$d.; £1 $2\frac{3}{5}$s. — $18\frac{1}{3}$s.; 2 yds. $1\frac{4}{9}$ ft. + $2\frac{5}{6}$ ft.
7. $\frac{129}{5} + \frac{88}{3}$; $\frac{19}{4} - 2\frac{2}{5}$; $\frac{127}{7} + 2\frac{1}{3} - \frac{97}{6}$; $9 - \frac{37}{9} + \frac{43}{6}$.

Answers. (1.) $3\frac{23}{24}$; $5\frac{13}{18}$; $\frac{1}{2}$. (2.) $4\frac{17}{42}$; $6\frac{7}{12}$. (3.) $2\frac{19}{20}$; $\frac{59}{72}$; $\frac{17}{28}$. (4.) 12s. $3\frac{2}{15}$d. (5.) $7\frac{1}{24}$d.; $11\frac{1}{4}$d. (6.) $9\frac{11}{20}$d.; $4\frac{4}{15}$s.; 3 yds. $1\frac{5}{18}$ ft. (7.) $55\frac{2}{15}$; $2\frac{7}{20}$; $4\frac{13}{42}$; $12\frac{1}{18}$.

Compound fractions must be reduced to simple fractions before the rule can be applied.

19. *To multiply fractions, or to find the fraction of a fraction*

Rule.—Multiply the numerators together for the new numerator and the denominators together for the new denominator. Thus $\frac{3}{4}$ of $\frac{2}{3}$ $=\frac{3 \times 2}{4 \times 3} = \frac{6}{12}$.

1st Proof. From the diagram Art. 13, we at once see that $\frac{1}{4}$ of $\frac{1}{3}$ is $\frac{1}{12}$, and that $\frac{1}{4}$ of $\frac{2}{3}$ is $\frac{2}{12}$; therefore 3 times $\frac{1}{4}$ of $\frac{2}{3}$ will be 3 times $\frac{2}{12}$ or $\frac{6}{12}$.

2nd Proof. To show that $\frac{4}{5}$ of $\frac{2}{3} = \frac{8}{15}$,
$\frac{15}{15} = 1$; dividing each by 3 (12), we get
$\frac{5}{15} = \frac{1}{3}$ of $1 = \frac{1}{3}$; dividing each by 5
$\frac{1}{15} = \frac{1}{5}$ of $\frac{1}{3}$.
Or generally $\frac{30}{15} = 2$; dividing each by 3 (9 and 12), we get
$\frac{10}{15} = \frac{1}{3}$ of $2 = \frac{2}{3}$; dividing each by 5
$\frac{2}{15} = \frac{1}{5}$ of $\frac{2}{3}$; multiplying each by 4
$\frac{8}{15} = 4$ times $\frac{1}{5}$ of $\frac{2}{3} = \frac{4}{5}$ of $\frac{2}{3}$.

3rd Proof. To show that $\frac{2}{7}$ of $\frac{3}{5} = \frac{6}{35}$. By Art. 13, $\frac{3}{5} = \frac{21}{35}$; then $\frac{2}{7}$ of $\frac{3}{5} = \frac{2}{7}$ of $\frac{21}{35} = 2$ times $\frac{3}{35} = \frac{6}{35}$.

The fraction of a fraction is equal to their product; thus $\frac{4}{5}$ of $\frac{2}{3} = \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

Proof. $\frac{2}{3} \times \frac{5}{5} = \frac{2}{3}$; taking the 5th of the multiplier, we get
$\frac{2}{3} \times \frac{1}{5} = \frac{1}{5}$ of $\frac{2}{3}$; taking the multiplier 4 times,
$\frac{2}{3} \times \frac{4}{5} = 4$ times $\frac{1}{5}$ of $\frac{2}{3} = \frac{4}{5}$ of $\frac{2}{3}$.

When there are mixed numbers in the compound fraction, it is generally most convenient to reduce the mixed numbers to improper fractions before multiplying.

Ex. 1. $\frac{3}{5}$ of $2\frac{1}{2}$ of $1\frac{2}{7}$ of $4 = \frac{3}{5} \times \frac{5}{2} \times \frac{9}{7} \times \frac{4}{1} = \frac{540}{70} = 7\frac{5}{7}$.
2. $2\frac{1}{4} \times 1\frac{2}{3}$ of $1\frac{1}{7} = \frac{9}{4} \times \frac{5}{3} \times \frac{8}{7} = \frac{360}{84} = 4\frac{2}{7}$.

In examples like this, the calculation is much simplified by striking out or *cancelling* any factors common to any one of the numerators and also any one of the denominators; thus

$$\frac{\overset{3}{\cancel{9}}}{\cancel{4}} \times \frac{5}{\cancel{3}} \times \frac{\overset{2}{\cancel{8}}}{7} = \frac{3 \times 5 \times 2}{7} = \frac{30}{7} = 4\frac{2}{7}.$$

3. $2\frac{13}{18}$ of $2\frac{6}{7}$ of $2\frac{2}{5} = \dfrac{\overset{7}{\cancel{49}}}{\underset{3}{\cancel{18}}} \times \dfrac{\overset{4}{\cancel{20}}}{\cancel{7}} \times \dfrac{\overset{2}{\cancel{12}}}{\cancel{5}} = \frac{56}{3} = 18\frac{2}{3}$.

4. $\frac{124}{3} + \frac{2}{3}$ of $\frac{3}{8} \times \frac{1}{6}$ of $4 = 41\frac{1}{3} + \frac{1}{6} = 41 + \frac{2}{6} + \frac{1}{6} = 41\frac{1}{2}$.

5. $\frac{7}{4} + \frac{2}{3}$ of $\frac{5}{4} - \frac{1}{6} \times \frac{7}{2} = 1\frac{3}{4} + \frac{5}{6} - \frac{7}{12} = 1 + \dfrac{9 + 10 - 7}{12} = 2$.

Exercises. Find the value of

1. $\frac{8}{35}$ of $2\frac{1}{4}$ of 70; $\frac{7}{32} \times 3\frac{1}{4} \times 4\frac{4}{7}$: $5\frac{1}{2} \times \frac{4}{5} \times 3\frac{3}{4}$; $3\frac{1}{4} \times 2\frac{1}{2} \times \frac{8}{5}$.
2. $\frac{4}{21} \times 8\frac{2}{5} \times 6\frac{2}{3}$; $\frac{1}{2}$ of $\frac{3}{4} - \frac{1}{6}$; $\frac{2}{7}$ of $2 \times 1\frac{3}{4}$; $\frac{19}{3} - \frac{1}{2}$ of 9.
3. $\frac{2}{5}$ of $\frac{3}{7} \times 3\frac{1}{8} \times 5\frac{1}{4}$; $2\frac{1}{3} \times 3\frac{1}{4} + \frac{1}{3}$ of $2\frac{1}{2}$; $\frac{9}{5} - \frac{1}{3}$ of $\frac{6}{7}$.
4. $8\frac{1}{2} \times \frac{1}{4} - \frac{7}{6}$; $2\frac{1}{2} \times 4\frac{1}{2} + \frac{3}{4}$ of $5\frac{1}{3} - \frac{1}{8}$; $5 - \frac{1}{5}$ of 9.
5. $\frac{2}{3} \times 6 \times \frac{1}{12} + \frac{7}{6}$ of $5 - \frac{1}{3}$ of $1\frac{1}{2}$; $\frac{10}{9} + \frac{1}{3}$ of $2\frac{1}{6} - \frac{5}{12}$ of 2.

Answers. (1.) 36; $3\frac{1}{4}$; $16\frac{1}{2}$; 13. (2.) $10\frac{2}{3}$; $\frac{5}{24}$; 1; $1\frac{5}{6}$. (3.) $2\frac{13}{16}$; $8\frac{5}{12}$; $1\frac{18}{35}$. (4.) $\frac{23}{24}$; $15\frac{1}{8}$; $3\frac{1}{5}$. (5.) $5\frac{2}{3}$; 1.

20. *Compound multiplication when there is a fraction in the multiplier.*

Ex. 1. What will be the cost of $\frac{7}{8}$ of a lb. of sugar at 2s. 8d. per lb.?

	s.	d.
	2	8
		7
8)	18	8
	2	4

	s.	d.
Cost 1 =	2	8
" $\frac{1}{8}$ =	0	4
" $\frac{7}{8}$ =	2	4

The second solution here given shows another way of multiplying by $\frac{7}{8}$, namely, by dividing the cost of 1 by 8 and subtracting the result from the cost of 1.

Ex. 2. What will be the cost of $3\frac{5}{8}$ when the cost of 1 is 1s. 8d.?

	s.	d.
	1	8
		3
	5	0
For $\frac{5}{8}$	1	$0\frac{1}{2}$
	6	$0\frac{1}{2}$

	s.	d.
Cost 1 =	1	8
		3
Cost 3 =		0
" $\frac{1}{2}$ =		10
" $\frac{1}{8}$ =		$2\frac{1}{2}$
" $3\frac{5}{8}$ =	6	$0\frac{1}{2}$

The value of $\frac{5}{8}$ in the first solution is found in the ordinary way, that is, by multiplying 1s. 8d. by 5 and dividing the result by 8; in the second solution the value of $\frac{4}{8}$ or $\frac{1}{2}$ is first found by dividing 1s. 8d. by 2, and this last result divided by 4 gives the value of $\frac{1}{8}$, then by adding the three results, we find the value of $3 + \frac{1}{2} + \frac{1}{8}$, or $3\frac{5}{8}$.

	£	s.	d.	
Ex. 3	1	18	$7\frac{1}{2}$	$\times\ 7\frac{7}{8}$
			8	
	15	9	0	
For $\frac{1}{8}$		4	$3\frac{1}{2}$	
	£15	4	$8\frac{1}{2}$	

Here we multiply by 8, but as this is $\frac{1}{8}$ too much we subtract $\frac{1}{8}$ of the price of 1 from this result for the product required.

	s.	d.	
Ex. 4.	8	$4\frac{7}{8}$	$\times\ 15$
		3	
1	5	$2\frac{5}{8}$	
		5	
£6	6	$1\frac{1}{8}$	

Here $\frac{7}{8} \times 3 = \frac{21}{8} = 2\frac{5}{8}$; we set down $\frac{5}{8}$d., and carry 2d. Again, $\frac{5}{8} \times 5 = \frac{25}{8} = 3\frac{1}{8}$; we set down $\frac{1}{8}$d., and carry 3d.

The calculation in this question may be simplified as follows. At once to get rid of the fraction we first multiply by 8 and then by 2, and subtract the value of 1 from the result for the value of 15; thus

	s.	d.	
	8	$4\frac{7}{8}$	
		8	
3	7	3	
		2	
6	14	6	product by 16
	8	$4\frac{7}{8}$	„ 1
£6	6	$1\frac{1}{8}$	„ 15

Ex. 5. Multiply £2 $5\frac{2}{5}$s. by $6\frac{2}{3}$; and 2 ft. $3\frac{1}{3}$ in. by $2\frac{2}{5}$.

In questions of this kind, it is generally best to reduce the quantity to one denomination before multiplying.

Here £2 $5\frac{2}{5}$s. $= 45\frac{2}{5}$s. $= \frac{227}{5}$s.; and $6\frac{2}{3} = \frac{20}{3}$; then $\frac{227}{5}$s. $\times \frac{20}{3} = 302\frac{2}{3}$s. $=$ £15 2s. 8d.

Again, 2 ft. $3\frac{1}{3}$ in. $= \frac{82}{3}$ in.; $2\frac{2}{5} = \frac{12}{5}$; then $\frac{82}{3}$ in. $\times \frac{12}{5} = 65\frac{3}{5}$ in. $=$ 5 ft. $5\frac{3}{5}$ in.

Ex. 6. Multiply 3 cwt. 23 lbs. by $8\frac{1}{3}$.

	cwt.	lbs.
	3	23
		8
	25	72
For $\frac{1}{3}$	1	$7\frac{2}{3}$
	26	$79\frac{2}{3}$

3 cwt. 23 lbs. $=$ 359 lbs.

	359 lbs.
	$8\frac{1}{3}$
	2872
For $\frac{1}{3}$	$119\frac{2}{3}$
112)	$2991\frac{2}{3}$ lbs.
	26 cwt. $79\frac{2}{3}$ lbs.

The first method here given is the better when the answer is required in cwts., and the second is preferable when the answer is required in lbs.

Ex. 7. Required the value of $\frac{3}{5}$ of a pound, $\frac{2}{3}$ of a shilling, and $\frac{5}{6}$ of 12s. 6d.

Here, $\frac{3}{5}$ of 20s. = 12s.; $\frac{2}{3}$ of 12d. = 8d.; and $\frac{5}{6}$ of 12s. 6d. = 10s. 5d. Then sum of these results = £1 3s. 1d.

Ex. 8. Find the sum of $\frac{4}{5}$ of £10 11s. 8d. and $\frac{8}{9}$ of £27 11s. 3d.

The following is more expeditious than the usual method of calculation.

	£	s.	d.
	10	11	8
Value of $\frac{1}{5}$ =	2	2	4
„ $\frac{4}{5}$ =	8	9	4
	24	10	0
	£32	19	4 *Ans.*

	£	s.	d.
	27	11	3
Value of $\frac{1}{9}$ =	3	1	3
„ $\frac{8}{9}$ =	24	10	0

Exercises. Find the value of

1. $\frac{7}{8}$ of £1; $\frac{7}{10}$ of £5; 8s. 4d. $\times \frac{2}{5}$; $2\frac{1}{4}$ of 2s. 6d.; $4\frac{1}{2}$s. $\times 3\frac{7}{9}$.
2. Find the difference of 3s. 6d. $\times 4\frac{2}{3}$, and 4s. $7\frac{1}{2}$d. $\times 2\frac{5}{6}$.
3. Find the sum and difference of £$2\frac{2}{3} \times 5\frac{3}{8}$ and £$3\frac{1}{7} \times 4\frac{2}{11}$.
4. 2 tons $\times \frac{3}{5}$ + 5 cwt. $\times 4\frac{3}{10}$; 3 cwt. 2 qr. $\times 2\frac{3}{4}$; 4 wk. 4d. $\times \frac{1}{4}$ of $8\frac{1}{2}$.
5. £4 9s. 6d. $\times 3\frac{2}{3}$ + 2s. 8d. $\times 4\frac{1}{8}$ — £4 7s. $\times 3\frac{2}{3}$.
6. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{8}$ of 3 cwt.; $\frac{3}{4}$ of 1 yd. + $\frac{2}{5}$ of 8 ft.; $\frac{3}{5}$ of 7 ft. — $\frac{3}{5}$ of 5 ft. 4 in.

Answers. (1.) 17s. 6d.; £3 10s.; 3s. 4d.; 5s. $7\frac{1}{2}$d.; 17s. (2.) 3s. $2\frac{3}{4}$d. (3.) £$27\frac{10}{21}$; £$1\frac{4}{21}$. (4.) 2 tons $5\frac{1}{2}$ cwt.; 9 cwt. $2\frac{1}{2}$ qr.; 9 wk. 5d. (5.) £1 0s. 2d. (6.) 7 lbs.; 1 yd. 2 ft. $5\frac{2}{5}$ in.; 1 ft.

21. *To divide fractions.*

Rule.—Invert the divisor and then multiply.

$$\text{Thus } \frac{3}{5} \div \frac{2}{7} = \frac{3}{5} \times \frac{7}{2} = \frac{21}{10}.$$

Proof. In all cases of division, *the divisor* $\times$ *the quotient* = *the dividend.*

In this case we have to divide $\frac{3}{5}$ by $\frac{2}{7}$,

$$\therefore \quad \frac{2}{7} \times \text{quotient} = \frac{3}{5}$$

that is, twice the seventh of the quotient is equal to $\frac{3}{5}$; therefore, taking the half of both sides of the equality, we get

$$\frac{1}{7} \times \text{quotient} = \frac{1}{2} \text{ of } \frac{3}{5}.$$

Now taking both sides of the equality seven times,

$$\text{quotient} = 7 \text{ times } \frac{1}{2} \text{ of } \frac{3}{5} = \frac{7}{2} \text{ of } \frac{3}{5} = \frac{21}{10},$$

where we invert the divisor, $\frac{2}{7}$, and then multiply.

Or thus. $\frac{3}{5} \div 2 = \frac{6}{10} \div 2 = \frac{3}{10}$, Art. 13.

Now by taking $\frac{1}{7}$ the divisor, the quotient will be increased 7 times.

$$\therefore \quad \frac{3}{5} \div \frac{2}{7} = 7 \text{ times } \frac{3}{10} = \frac{21}{10}.$$

As another illustration of the principle, let us take the following example:—

Ex. 1. If $3\frac{6}{7}$ articles cost 18s., what will 1 article cost?

Here, Cost $3\frac{6}{7}$ = 18s.

Now, to get rid of the fraction, we shall multiply by 7. It is obvious that 7 times the number will cost 7 times as much. But it will be observed (Art. 12) that 7 times $3\frac{6}{7}$ = 27.

$\therefore$ Cost 27 = 18s. $\times$ 7

$\therefore$ Cost 1 = $\dfrac{18\text{s.} \times 7}{27}$ = $\frac{126}{27}$s. = 4s. 8d.

The method here given is often highly convenient in practice.

2. If $27\frac{5}{6}$ articles cost £106 9s. 3d., what is the cost of 1 article?

$$\begin{array}{rl} & \text{£} \quad \text{s.} \quad \text{d.} \\ 27\frac{5}{6}\ ) & 106 \quad 9 \quad 3\ (\\ 6 & \qquad\quad\ \ 6 \\ \hline 167\ ) & 638 \quad 15 \quad 6\ (\ \text{£}3\ 16\text{s.}\ 6\text{d.} \end{array}$$

Here to get rid of the fraction in the divisor, we multiply by 6, and then proceed with the division as in common long division. If we take the number of articles 6 times, we must increase the price 6 times.

3. £2 $5\frac{1}{7}$s. $\div 3\frac{2}{3}$ = $45\frac{1}{7}$s. $\div \frac{11}{3}$ = $\frac{316}{7}$s. $\times \frac{3}{11}$ = $\frac{948}{77}$s. = 12s. $3\frac{57}{77}$d.

4. $8\frac{3}{4} \div 3\frac{1}{2} \times \frac{1}{5} = \frac{35}{4} \div \frac{7}{2} \times \frac{6}{5} = \frac{35}{4} \times \frac{2}{7} \times \frac{5}{6} = 2\frac{1}{12}$.

5. $\frac{5}{8} \times 2\frac{2}{3} \div \frac{1}{4}$ of $\frac{5}{7} = \frac{5}{8} \times \frac{8}{3} \times \frac{4}{1} \times \frac{7}{5} = \frac{28}{3} = 9\frac{1}{3}$.

Exercises. $\frac{3}{8} \div \frac{5}{4}$; $1\frac{2}{5} \div \frac{3}{10}$; $2\frac{1}{3} \div 1\frac{3}{4}$; £3 $2\frac{1}{8}$s. $\div 2\frac{3}{4}$; £$8\frac{1}{3} \div 8\frac{8}{9}$; £$2\frac{2}{3} \div 1\frac{7}{9}$; £3 6s. 4 $\div 24\frac{7}{8}$; $\frac{7}{8}$ of $2\frac{2}{3} \div 1\frac{2}{7}$ of $2\frac{1}{3}$; $1\frac{2}{3} \div \frac{3}{5}$ of $1\frac{1}{2}$ of $6\frac{2}{3}$; $\frac{3}{7} \times 2\frac{1}{3} \div 3\frac{2}{3}$; $1\frac{1}{2} \div 2\frac{1}{2} + 1\frac{1}{3} \div 1\frac{2}{3}$; $\frac{2}{3}$ of $7\frac{1}{2}$ of $\frac{3}{10} \div 1\frac{1}{2}$; $\frac{3}{4}$ of £1 $\div \frac{3}{4}$ of 1s.; $\frac{2}{3}$ of £32 $\div 1\frac{3}{5}$; $\frac{2}{3}$ of £$3\frac{3}{4} \div \frac{5}{3}$ of £$2\frac{1}{4}$.

Answers. $\frac{3}{10}$; $4\frac{2}{3}$; $1\frac{1}{3}$; £1 2s. $7\frac{1}{11}$d.; 18s. 9d.; £$1\frac{1}{2}$; 2s. 8d.; $\frac{7}{9}$; $\frac{5}{12}$; $\frac{1}{3}$; $1\frac{2}{5}$; 1; 20; £$13\frac{1}{3}$; $\frac{2}{3}$.

22. *To change the denomination of a quantity, or to express a given quantity as the fraction of another.*

Ex. 1. Reduce 3s. 6d. to the fraction of £1; and $6\frac{3}{4}$ ft. to yards.

$$3\text{s. } 6\text{d.} = 42\text{d.} = \text{£}\,\frac{42}{12 \times 20} = \text{£}\tfrac{7}{40}.$$

$$6\tfrac{3}{4}\text{ ft.} = \tfrac{27}{4}\text{ ft.} = \frac{27}{4 \times 3}\text{ yds.} = \tfrac{9}{4}\text{ yds.} = 2\tfrac{1}{4}\text{ yds.}$$

2. Express 2s. 6d. as the fraction of 5s. 4d.; and $1\frac{3}{4}$d. as the fraction of 1s.

2s. 6d. = 30d.; 5s. 4d. = 64d.;

Then the fraction required = $\frac{30}{64} = \frac{15}{32}$.

Again, $1\frac{3}{4}$d. = $\frac{7}{4}$d. = $\dfrac{7}{4 \times 12}$s. = $\frac{7}{48}$s.

3. Express $2\frac{2}{10}$ lbs. as a fraction of $5\frac{1}{2}$ lbs.

Here, the fraction required = $\dfrac{2\frac{2}{10}}{5\frac{1}{2}} = \frac{22}{55} = \frac{2}{5}$.

4. Reduce $\frac{2}{3}$ of 5s. to the fraction of £1; and $\frac{2}{7}$ of 5 cwt. 1 qr. to the fraction of a ton.

$\frac{2}{3}$ of 5s. = $\frac{10}{3}$s. = $\dfrac{10}{3 \times 20}$£ = £$\frac{1}{6}$.

$\frac{2}{7}$ of 5 cwt. 1 qr. = $\frac{2}{7}$ of $\frac{21}{4}$ cwt. = $\frac{2}{7} \times \frac{21}{4} \times \frac{1}{20}$ tons = $\frac{3}{40}$ tons.

5. Find the value of $\frac{5}{8}$ of a pound + $\frac{3}{4}$ of a shilling — $\frac{1}{2}$ of a guinea.

$$£\tfrac{5}{8} = \frac{5 \times 20}{8}\text{s.} = \tfrac{25}{2}\text{s.};\ \text{then } \tfrac{25}{2}\text{s.} + \tfrac{3}{4}\text{s.} - \tfrac{21}{2}\text{s.} = 2\tfrac{3}{4}\text{s.} = 2\text{s. } 9\text{d.}$$

Exercises. Reduce

1. 6s. 3d.; 1s. $4\frac{1}{4}$d.; 1s. $2\frac{1}{8}$d.; £2 11s. 3d.; £1 2s.; £3 0s. 1d.; to the fraction of £1. *Ans.* $\frac{5}{16}$; $\frac{13}{192}$; $\frac{113}{1920}$; $\frac{41}{16}$; $\frac{11}{10}$; $\frac{721}{240}$.

2. $1\frac{1}{4}$d.; $\frac{1}{4}$d.; $\frac{1}{3}$d.; $3\frac{3}{8}$d.; 1s. 3d.; 9s. 2d; to the fraction of a shilling. *Ans.* $\frac{5}{48}$; $\frac{1}{48}$; $\frac{1}{36}$; $\frac{9}{32}$; $\frac{5}{4}$; $\frac{55}{6}$.

3. $22\frac{2}{5}$ lbs.; 2 qrs. 14 lbs.; 1 cwt. 3 qrs. 21 lbs.; 2 qrs. 7 lbs; $\frac{2}{3}$ of $\frac{5}{6}$ of $28\frac{4}{5}$ lbs.; to the fraction of a cwt. *Ans.* $\frac{1}{5}$; $\frac{5}{8}$; $\frac{31}{16}$; $\frac{9}{16}$; $\frac{1}{7}$.

4. $6\frac{1}{2}$ hours; 2 days $3\frac{2}{5}$ hours; 3 weeks $6\frac{3}{4}$ hours; $1\frac{1}{2}$ days $4\frac{1}{4}$ hours; to the fraction of a day. *Ans.* $\frac{13}{48}$; $\frac{257}{120}$; $\frac{681}{32}$; $\frac{161}{96}$.

5. $\frac{3}{5}$s.; $2\frac{1}{4}$d.; 3s. $6\frac{2}{5}$d.; £2 16s.; £$6\frac{3}{5}$; to the fraction of 7s. 6d. *Ans.* $\frac{2}{25}$; $\frac{1}{40}$; $\frac{106}{225}$; $\frac{112}{15}$; $\frac{88}{5}$.

6. 9s. 6d.; $\frac{7}{8}$s.; £$\frac{4}{5}$; $8\frac{2}{5}$s.; $2\frac{1}{3}$d.; to the fraction of a guinea. *Ans.* $\frac{19}{42}$; $\frac{1}{24}$; $\frac{16}{21}$; $\frac{2}{5}$; $\frac{1}{108}$.

7. $\frac{6}{7}$ of $1\frac{1}{2}$d.; $\frac{8}{9}$ of $5\frac{1}{4}$d.; $\frac{4}{5}$ of $3\frac{4}{7}$d., to the fraction of a shilling; and $\frac{6}{7}$ of 6d; $\frac{4}{5}$ of $1\frac{3}{4}$s. to the fraction of £1. *Ans.* $\frac{3}{28}$; $\frac{7}{18}$; $\frac{5}{21}$; $\frac{3}{140}$; $\frac{7}{100}$.

8. Find the value of £$\frac{7}{8}$ — £$\frac{2}{3}$ + $\frac{5}{6}$s. — $\frac{1}{8}$ of 1 guinea. *Ans.* $2\frac{3}{8}$s.

9. £$2\frac{1}{10}$ — $3\frac{7}{8}$s. — $1\frac{1}{2}$d. = £$\frac{19}{10}$; $2\frac{7}{8}$ lbs. + $4\frac{8}{9}$ oz. — $1\frac{3}{4}$ lbs. = $\frac{103}{72}$ lbs.

10. $\frac{3}{5}$ hour + $\frac{1}{4}$ week + $\frac{5}{8}$ day = $\frac{12}{5}$ days; $\frac{5}{8}$ cwt. — $3\frac{1}{2}$ lbs. = $\frac{19}{32}$ cwt.

23. *To calculate complex fractions.*

1. $\frac{2}{3} \times 3\frac{2}{5} \times 5 - \frac{3}{7} \text{ of } \frac{5}{6} \div 1\frac{5}{7} + \frac{1}{9} \div 1\frac{1}{9}.$
$= \frac{2}{3} \times 17 - \frac{3}{7} \times \frac{5}{6} \times \frac{7}{12} + \frac{1}{10} = 11\frac{1}{3} - \frac{5}{24} + \frac{1}{10} = 11\frac{9}{40}.$

2. $\frac{2}{3}(5 + \frac{5}{2})(2 - \frac{2}{5}) = \frac{2}{3} \times \frac{15}{2} \times \frac{8}{5} = 8.$

3. $$\frac{1 + \frac{5}{6}}{1 - \frac{3}{4}} = \frac{12 + 10}{12 - 9} = \tfrac{22}{3} = 7\tfrac{1}{3}.$$

Here to get rid of the fractions, we first multiply numerator and denominator by 12, which is the L. C. M. of 6 and 4.

4. $$\frac{\frac{5}{6}}{\frac{2}{9}} = \tfrac{5}{6} \div \tfrac{2}{9} = \tfrac{5}{6} \times \tfrac{9}{2} = \tfrac{15}{4} = 3\tfrac{3}{4}.$$

Here we have a simple fraction in the numerator and also in denominator; in such cases the fraction is simplified by multiplying the two *outer* numbers for the new numerator, and the two *inner* numbers for the new denominator.

5. $\frac{7}{5}(1 + \frac{5}{6} \text{ of } \frac{2}{7}) - \frac{5}{4}(3 - \frac{4}{5} \text{ of } 2\frac{1}{2}) = \frac{7}{5} \times 1\frac{5}{21} - \frac{5}{4} \times 1 = \frac{29}{60}.$

6. $$\frac{8 \times 7 + 4 \times 2}{8 \times 9 - 4 \times 7} = \frac{2 \times 7 + 1 \times 2}{2 \times 9 - 1 \times 7} = \tfrac{16}{11} = 1\tfrac{5}{11}.$$

Here, to simplify the calculation, we first divide each part of the numerator and denominator by 4. See Axiom 8, Art. 2.

7. $3\frac{3}{5} \div (\frac{2}{3} \text{ of } 5\frac{2}{5} \div \frac{5}{9}) = \frac{18}{5} \div (\frac{18}{5} \times \frac{9}{5}) = \frac{18}{5} \times \frac{5}{18} \times \frac{5}{9} = \frac{5}{9}.$

8. $2\frac{1}{2} \div 1\frac{3}{4} + 4\frac{2}{3} \div 7 + (3 \div 2\frac{1}{2}) \times (4 \div 1\frac{1}{3}) \div (12 \div 1\frac{1}{4})$
$= \frac{5}{2} \times \frac{4}{7} + \frac{14}{3} \times \frac{1}{7} + 3 \times \frac{2}{5} \times 4 \times \frac{3}{4} \times \frac{5}{48} = 1\frac{1}{7} + \frac{2}{3} + \frac{3}{8} = 2\frac{31}{168}.$

9. Find the product of the sum and difference of $2\frac{1}{2}$ and $2\frac{1}{3}$.
$$(2\tfrac{1}{2} + 2\tfrac{1}{3}) \times (2\tfrac{1}{2} - 2\tfrac{1}{3}) = 4\tfrac{5}{6} \times \tfrac{1}{6} = \tfrac{29}{36}.$$

10. Add together the sum, difference, and product of $\frac{7}{12}$ and $\frac{4}{7}$.

$$(\tfrac{7}{12} + \tfrac{4}{7}) + (\tfrac{7}{12} - \tfrac{4}{7}) + \tfrac{7}{12} \times \tfrac{4}{7} = \tfrac{14}{12} + \tfrac{1}{3} = 1\tfrac{1}{2}.$$

Exercises.

1. $\dfrac{\frac{3}{4} \times 5}{\frac{5}{8} \times 3} = 2$; $\dfrac{\frac{3}{8}}{\frac{5}{6}} - \frac{1}{5}$ of $\frac{1}{2} = \frac{7}{20}$; $\dfrac{1 - \frac{2}{9}}{1 - \frac{5}{6}} \times \dfrac{1 - \frac{2}{3}}{1 - \frac{1}{6}} = 1\frac{13}{15}$.
2. $2 - \frac{3}{5} \times \frac{5}{4} \div 2\frac{2}{3} + 7\frac{1}{2} \div 4\frac{1}{2} - 5\,(\frac{1}{3} - \frac{1}{16}) = 2.$
3. $\dfrac{24 \times 8 - 16 \times 7}{8 \times 6 + 32 \times 3} = \frac{5}{9}$; $\dfrac{3\frac{3}{8}}{13\frac{1}{2}} = \frac{1}{4}$; $\dfrac{2\frac{1}{3}}{1\frac{1}{2}} \div \dfrac{4\frac{2}{3}}{1\frac{1}{5}} = \frac{2}{5}$.
4. $(2\frac{1}{2} \div 1\frac{3}{4}) \times (4\frac{2}{3} \div 6\frac{1}{4}) = 1\frac{1}{15}$; $(\frac{4}{3} \div 2\frac{2}{3}) \div (\frac{9}{2} \div \frac{8}{3}) = \frac{8}{27}$.
5. Find the product of the sum and difference of $5\frac{1}{3}$ and $4\frac{1}{4}$. *Ans.* $10\frac{55}{144}$.
6. Divide the product of $\frac{4}{5}$ and $\frac{5}{8}$ by their difference. *Ans.* $2\frac{6}{7}$.
7. Add together the sum and difference of $5\frac{1}{3}$ and $2\frac{4}{5}$, and divide the result by $\frac{2}{5}$ of $5\frac{1}{3} \div 2\frac{4}{5}$. *Ans.* 14.

24. *Problems in fractions.*

1. If $\frac{2}{3}$ of $\frac{3}{5}$ of property be worth £260, what is the value of the whole?

$\frac{2}{3}$ of $\frac{3}{5} = \frac{2}{5}$; $\therefore$ Value of $\frac{2}{5}$ = £260.
$\therefore$ " $\frac{1}{5} = \frac{1}{2}$ of £260 = £130.
$\therefore$ " 1 = 5 times £130 = £650.

2. If $\frac{5}{6}$ be worth £20, what is the value of the whole? *Ans.* £24.
3. $\frac{1}{2}$ of $\frac{1}{5}$ of a number is $1\frac{7}{10}$, what is the number? *Ans.* 17.
4. $\frac{1}{3} + \frac{1}{4} + \frac{1}{8}$ of a number is 34, what is the number?

$$\tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{8} = \frac{8 + 6 + 3}{24} = \tfrac{17}{24};\ \text{then}$$

$\frac{17}{24}$ of the number = 34,
$\therefore$ $\frac{1}{24}$ " $= \frac{34}{17} = 2.$
$\therefore$ the number = 24 times 2 = 48.

5. $\frac{1}{2} + \frac{1}{3}$ of a number is 10, what is the number? *Ans.* 12.
6. If $\frac{2}{3}$ of an estate be worth £50, find the value of $\frac{3}{5}$. *Ans.* £45.
7. What number divided by $7\frac{2}{5}$ will give $18\frac{1}{2}$? *Ans.* $136\frac{9}{10}$.
8. How often is £$\frac{3}{5}$ contained in 4 guineas? *Ans.* 7 times.
9. What part of a lb. are $3\frac{1}{5}$ oz.? *Ans.* $\frac{1}{5}$.
10. Divide £5 between A and B, so that A shall have $1\frac{1}{2}$ times as much as B.

Supposing B to have 1 share, A will have $1\frac{1}{2}$ shares; and then dividing the money into $2\frac{1}{2}$ shares, we have—

$2\frac{1}{2}$ shares = £5.
$\therefore$ 1 share = £5 $\div 2\frac{1}{2}$ = £2, B's; $\therefore$ A's = £5 — £2 = £3.

11. Divide 5s. between A and B, so that A shall have $2\frac{1}{3}$ times as much as B. *Ans.* 1s. 6d. A's share; 3s. 6d. B's share.
12. A person spent $\frac{1}{3}$ of his money, and afterwards $\frac{3}{4}$ of the remainder, and then found that he had $2\frac{1}{3}$s. left; how much had he at first?

Part spent at first $= \frac{1}{3}$; Part spent afterwards $= \frac{3}{4}$ of $(1 - \frac{1}{3}) = \frac{1}{2}$;

$\therefore$ Part spent altogether $= \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$;

$\therefore$ Part remaining $= 1 - \frac{5}{6} = \frac{1}{6}$;

$\therefore \frac{1}{6}$ of his money $= 2\frac{1}{3}$s.; $\therefore$ his money $= 2\frac{1}{3}\text{s.} \times 6 = 14$s.

13. A man sold $\frac{1}{5}$ of his oranges to A, $\frac{1}{4}$ of the remainder to B, and had 9 left; how many had he at first? *Ans.* 15.

14. How many oranges did B purchase? *Ans.* 3.

15. A person left £240 among three persons; to A he left $\frac{3}{8}$, to B $\frac{1}{6}$ of A's share, and to D the rest; required D's share of the money.

A's share $= \frac{3}{8}$; B's share $= \frac{1}{6}$ of $\frac{3}{8} = \frac{1}{16}$;

$\therefore$ D's share $= 1 - \frac{3}{8} - \frac{1}{16} = \frac{9}{16}$;

$\therefore$ Value D's share $= \frac{9}{16}$ of £240 $=$ £135.

Or thus. Value A's share $= \frac{3}{8}$ of £240 $=$ £90.

" B's " $= \frac{1}{6}$ of £90 $=$ £15.

" D's " $=$ £240 $-$ £90 $-$ £15 $=$ £135.

16. Find the value of D's share, when B's is $\frac{1}{4}$ of A's? *Ans.* £$127\frac{1}{2}$.

17. Out of $\frac{3}{4}$ of $\frac{16}{21}$ of £63, I paid £30; how much remains? *Ans.* £6.

18. Divide $\frac{7}{8}$ of $\frac{4}{9}$ of £$\frac{1}{3}$ by $\frac{3}{4}$ of $\frac{7}{16}$. *Ans.* 7s. $10\frac{22}{27}$d.

19. What number multiplied by $\frac{5}{6}$ of $\frac{9}{10}$ is equal to $5\frac{2}{3}$?

$\frac{5}{6}$ of $\frac{9}{10} = \frac{3}{4}$; the number $\times \frac{3}{4} = 5\frac{2}{3}$; $\therefore$ the number $= 5\frac{2}{3} \div \frac{3}{4} = 7\frac{5}{9}$.

20. What number divided by $\frac{3}{5}$ of $\frac{1}{27}$ of 10 will give $2\frac{1}{4}$? *Ans.* $\frac{1}{2}$.

21. What number added to $\frac{2}{3}$ of $1\frac{1}{2}$ of $\frac{5}{8}$ will amount to unity? *Ans.* $\frac{3}{8}$.

22. 2s. 6d. are how many times $5\frac{1}{2}$d.; 480 yards are what part of a mile; 8 cwt. 14 lbs. are how many times 3 cwt. 1 qr.?

Ans. $5\frac{5}{11}$; $\frac{3}{11}$; $2\frac{1}{2}$

23. If a man does $\frac{3}{8}$ of a piece of work in $4\frac{1}{2}$ hours, what part of it would he do in 1 hour, and in what time would he do the whole?

No. hours to do $\frac{3}{8} = 4\frac{1}{2} = \frac{9}{2}$.

$\therefore$ " $\frac{1}{8} = \frac{1}{3}$ of $\frac{9}{2} = \frac{3}{2}$.

$\therefore$ " $1 = 8$ times $\frac{3}{2} = 12$.

And, therefore, the part done in 1 hour $= \frac{1}{12}$.

24. In what time would he finish the remainder? *Ans.* $7\frac{1}{2}$ hours.

25. If A can do a piece of work in 9 hours, B in 8 hours, and C in 6 hours; in what time can they do it working together?

In 1 hour, A does $\frac{1}{9}$, B $\frac{1}{8}$, and C $\frac{1}{6}$:

$\therefore$ Part done by A, B, and C in 1 hour $= \frac{1}{9} + \frac{1}{8} + \frac{1}{6} = \frac{29}{72}$;

that is, A, B, and C do $\frac{29}{72}$ of the whole in one hour; then

No. hours to do $\frac{29}{72}$ of the whole $= 1$;

$\therefore$ " $\frac{1}{72}$ " $= \frac{1}{29}$;

$\therefore$ " the whole $= 72$ times $\frac{1}{29} = 2\frac{14}{29}$.

26. In what time would A and B, working together, do it?

Ans. $4\frac{4}{17}$ hours.

27. Suppose A to work alone for 2 hours, and then B alone for 1 hour, in what time would A and B finish the remainder?

Part done by A and B working separately $= \frac{2}{9} + \frac{1}{8} = \frac{25}{72}$;

Part remaining to be done $= 1 - \frac{25}{72} = \frac{47}{72}$;

and proceeding, as in the foregoing examples, we find

No. hours for A and B to do $\frac{1}{72}$ part $= \frac{1}{17}$:

$\therefore$ " " $\frac{47}{72}$ " $= 47$ times $\frac{1}{17} = 2\frac{13}{17}$.

28. A cistern can be filled in 16 minutes by one pipe (A), and in 12 minutes by another pipe (B). (1.) in what time will they fill it when opened together? (2.) if A alone be opened for 4 minutes, in what time will they fill the remaining portion of the cistern when opened together?
Ans. $6\frac{6}{7}$, $5\frac{1}{7}$ minutes.

29. If the pipe A discharges the water (see last example) *from* the cistern, in what time would the cistern be filled when the pipes are opened together?

In this case, the part filled per minute $= \frac{1}{12} - \frac{1}{16} = \frac{1}{48}$; therefore, the whole must be filled in 48 minutes.

30. A can do a piece of work in 15 days, A and B working together can do it in 9 days, and A and C in 8 days; in what time would B and C do it?

In 1 day, A can do $\frac{1}{15}$, A and B $\frac{1}{9}$, and A and C $\frac{1}{8}$;
∴ The part done by B in 1 day $= \frac{1}{9} - \frac{1}{15} = \frac{2}{45}$; and
„ C „ $= \frac{1}{8} - \frac{1}{15} = \frac{7}{120}$.
∴ „ B and C „ $= \frac{2}{45} + \frac{7}{120} = \frac{37}{360}$.
∴ No. days for B and C to do the whole $= \frac{360}{37} = 9\frac{27}{37}$.

31. If A can mow a field in 12 days, and A and B working together can do it in 5 days, in what time would B alone do it? *Ans.* $8\frac{4}{7}$ days.

32. In what time would A, B, and C (Example 30) do the whole?
Ans. $5\frac{55}{61}$ days.

33. A man travelled $16\frac{1}{2}$ miles in $3\frac{3}{4}$ hours; at what rate did he travel per hour?

No. miles per hour $= 16\frac{1}{2} \div 3\frac{3}{4} = \frac{33}{2} \times \frac{4}{15} = 4\frac{2}{5}$.

34. If $9\frac{3}{4}$ lbs. of tea cost 21s. 9d., what is the price per lb.?
Ans. 2s. $2\frac{10}{13}$d.

35. A man finished a piece of work in 6 days; in the first 2 days he worked $10\frac{3}{4}$ hours each day, in the succeeding 3 days $9\frac{1}{4}$ hours each day, and in the last day 7 hours 5 minutes; how many hours per day should he work to finish the whole in 5 days?

No. hours work in the whole $= 10\frac{3}{4} \times 2 + 9\frac{1}{4} \times 3 + 7\frac{1}{12} = 56\frac{1}{3}$.
∴ No. hours required $= 56\frac{1}{3} \div 5 = 11$ h. 16 min.

36. A man completes a journey in 4 days; in the first day he travelled for $11\frac{1}{4}$ hours, in the two succeeding days 9 hours 10 minutes each day, and in the last day 6 hours 20 minutes; how many hours per day should he travel to complete the same journey in 3 days? *Ans.* $11\frac{35}{36}$.

37. A watch gains 6 minutes in 12 hours. If it be set right at noon, what will be the true time when the watch is at 20 minutes past 7?

7 h. 20 min. = 440 min. past noon; gain per hour $= \frac{6}{12} = \frac{1}{2}$ min.;
∴ True time when the watch indicates $60\frac{1}{2}$ min. past noon = 1.

∴ „ „ 440 „ $= \dfrac{440}{60\frac{1}{2}}$
$= 7$ h. $16\frac{44}{121}$m.

38. What would be the true time when the watch gains 3 minutes in 12 hours? *Ans.* 7h. $18\frac{42}{241}$ min.

39. How much copper can be got from 1 ton of the ore, allowing that it yields $14\frac{1}{4}$ per cent. of pure copper?

Weight of copper in 100 tons $= 14\frac{1}{4}$ tons.
∴ „ „ 1 „ $= 14\frac{1}{4}$ tons $\div 100 = 319\frac{1}{5}$ lbs.

40. If I gain £$8\frac{5}{6}$ per £100, how much is that in £1? *Ans.* 1s. $9\frac{1}{5}$d.

41. A butcher bought 10 sheep and 5 lambs for £$10\frac{1}{2}$. Now each sheep cost 3 times as much as each lamb. What did he pay for each lamb?

Here, we shall turn the sheep into lambs.
10 sheep = 3 times 10 or 30 lambs;
∴ 10 sheep + 5 lambs = 30 lambs + 5 lambs = 35 lambs;
∴ Value 35 lambs = £$10\frac{1}{2}$; and value 1 lamb = £$10\frac{1}{2} \div 35$ = 6s.

42. A farmer bought 2 horses and 6 cows for £140; but the price of each horse was twice the price of each cow: required the price of each horse. *Ans.* £28.

43. A horse and chaise are worth £60, of which the chaise is $\frac{3}{5}$; required the value of the horse. *Ans.* £24.

44. A post is 2 ft. 4 in. in the ground, and $\frac{5}{7}$ of its whole length above the ground; required the whole length of the post. *Ans.* 8 ft. 2 in.

45. A horse and saddle cost £$21\frac{1}{3}$; but the horse cost £9 more than the saddle; required the price of the saddle.

As the horse cost £9 more than the saddle,
The value of 2 saddles = £$21\frac{1}{3}$ — £9 = £$12\frac{1}{3}$;
∴ " 1 " = £$12\frac{1}{3} \div 2$ = £6 3s. 4d.

46. Divide 9s. between two persons, so that one may have 4d. more than the other. *Ans.* 4s. 8d.; 4s. 4d.

47. A and B start a journey at the same time, and from the same place, in opposite directions; A travels at the rate of $3\frac{3}{4}$ miles an hour, and B $2\frac{1}{2}$ miles an hour. In what time will they be 40 miles apart?

Rate per hour at which they recede from each other = $3\frac{3}{4} + 2\frac{1}{2}$
= $6\frac{1}{4}$ miles; ∴ No hours required = $40 \div 6\frac{1}{4} = 6\frac{2}{5}$.

48. What distance would they be apart in $5\frac{1}{2}$ hours? *Ans.* $34\frac{3}{8}$ miles.

49. Supposing the men (Ex. 47) to travel in the *same* direction, in what time would they be 20 miles apart? *Ans.* 16 hours.

50. How many persons may receive each $1\frac{11}{12}$s. out of £$6\frac{9}{10}$? *Ans.* 72.

51. A person engaged a labourer on condition of paying him 2s. for every day he should work, and of charging 9d. for his board every day he should be idle. Now at the end of 4 weeks, or 24 days, the labourer was entitled to 37s. How many days did he work?

If he had worked for the whole time, he would have been entitled to 2s. × 24 = 48s.; therefore, the loss from idleness = 48s. — 37s. = 11s.; but the loss from one idle day = 2s. + 9d. = $2\frac{3}{4}$s.;
∴ No. idle days = 11s. ÷ $2\frac{3}{4}$s. = $11 \div 2\frac{3}{4} = 11 \times \frac{4}{11} = 4$;
∴ No. working days = 24 — 4 = 20.

52. Required the same as in the last example, when the wages are 2s. 8d. per day, the charge for board 10d., and the sum due 43s. *Ans.* 18.

DECIMAL FRACTIONS.

25. In decimals, or decimal fractions, the common scale of integer notation is carried below unity, a point being placed where the fractional parts begin. Thus 23·46 reads 2 tens + 3 units + 4 tenths + 6 hundredths, or $20 + 3 + \frac{4}{10} + \frac{6}{100}$. This extension of the common

numeration scale is only another way of writing down tenths, hundredths, thousandths, &c. It enables us to apply the four simple rules of integers to fractional expressions.

26. Any decimal fraction may be written down in the ordinary form of a fraction by simply writing its figures for the numerator, and for the denominator writing 10, 100, 1000, &c., according to the number of decimal figures.

Thus $4{\cdot}25 = \frac{425}{100}$; $\cdot025 = \frac{25}{1000}$; $\cdot45 = \frac{45}{100} = \frac{9}{20}$.

Proof. $4{\cdot}25 = 4 + \frac{2}{10} + \frac{5}{100} = \frac{400}{100} + \frac{20}{100} + \frac{5}{100} = \frac{425}{100}$.

$\cdot025 = \frac{2}{100} + \frac{5}{1000} = \frac{20}{1000} + \frac{5}{1000} = \frac{25}{1000}$.

27. Any fraction having 10, 100, &c., for its denominator, may be written down as a decimal by simply setting down the numerator, and marking off by the decimal point as many figures, from the right, as there are ciphers in the denominator, prefixing ciphers if necessary.

Thus $\frac{364}{100} = 3{\cdot}64$; and $\frac{32}{10000} = \cdot0032$.

Proof. $\frac{364}{100} = \frac{300}{100} + \frac{60}{100} + \frac{4}{100} = 3 + \frac{6}{10} + \frac{4}{100} = 3{\cdot}64$.

$\frac{32}{10000} = \frac{30}{10000} + \frac{2}{10000} = \frac{3}{1000} + \frac{2}{10000} = \cdot0032$.

28. A decimal is multiplied by 10, 100, &c., by moving the point one, two, three, &c., places to the *right*; and on the contrary a decimal is divided by 10, 100, 1000, &c., by moving the point one, two, three, &c., places to the *left*.

Thus $\cdot346 \times 100 = 34{\cdot}6$; and $\cdot346 \div 100 = \cdot00346$.

Proof. $\cdot346 \times 100 = \frac{346}{1000} \times 100 = \frac{346}{10} = 34{\cdot}6$. By Arts. 20 and 21.

$\cdot346 \div 100 = \frac{346}{1000} \div 100 = \frac{346}{100000} = \cdot00346$.

Placing ciphers *after* a decimal does not alter its value; thus ·3 is obviously the same as ·30. But placing ciphers *before* a decimal decreases its value, and is equivalent to dividing by 10, 100, &c.; thus $\cdot346 \div 100 = \cdot00346$.

29. *To add and subtract decimals.*

Rule. We add and subtract decimals in the same manner as common integers, observing to place the units' figures under one another, so that the decimal points may be in the same vertical column.

1. Add together 3·87, 2·68, ·09.

$$\begin{array}{r} 3{\cdot}87 \\ 2{\cdot}68 \\ \cdot09 \\ \hline 6{\cdot}64 \end{array}$$

Here the addition of the hundredths gives $\frac{24}{100} = \frac{2}{10} + \frac{4}{100}$; we, therefore, put 4 in the hundredths' place, and carry the 2 tenths to the column of tenths, and so on as in common addition.

2. Subtract 2·564 from 5·132.

$$\begin{array}{r} 5{\cdot}132 \\ 2{\cdot}564 \\ \hline 2{\cdot}568 \end{array}$$

Here, as we cannot take 4 thousandths from 2 thousandths, we borrow 1 hundredth or 10 thousandths from the 3 hundredths, and then 4 thousandths from 12 thousandths, and 8 thousandths remain. We have now to

take 6 hundredths from 2 hundredths, or, as in common subtraction, 7 hundredths from 3 hundredths, and so on as in common subtraction.

Exercises.

1. ·003 + 32·025 + 21·472 ; 21 + ·05 + 3·842 + ·018.
2. 2·574 + 3·069 + ·099 + 1·782 + ·99 ; 3·1 + ·8 + ·72.
3. 60 + 1·005 + ·08 + ·95 + ·845 ; 35·64 + ·06 + ·3.
4. 3 — ·04 ; 1 — ·002 ; 11·1 — 2·03 ; ·045 — .03.
5. 1·4 — ·805 ; 36·1 — 5·6 ; 61 — ·273 ; 1·27 — ·87.

Answers. (1.) 53·5; 24·9. (2.) 8·514 ; 4·62. (3.) 62·88 ; ·36. (4.) 2·96 ; ·998; 9·07 ; ·015. (5.) ·595 ; 30·5 ; 60·727 ; ·4.

30. *To multiply and divide decimals.*

Rule..To multiply decimals. Multiply as in common whole numbers, and mark off in the product as many decimal places as there are in the multiplier and multiplicand together.

1. Multiply 2·042 by 2·3.

$$\begin{array}{r} 2{\cdot}042 \\ 2{\cdot}3 \\ \hline 6126 \\ 4084 \\ \hline 4{\cdot}6966 \end{array}$$

Here there are *three* decimal places in the multiplicand and *one* in the multiplier, therefore we mark off *four* in the product.

Proof. $2{\cdot}042 \times 2{\cdot}3 = \frac{2042}{1000} \times \frac{23}{10} = \frac{46966}{10000} = 4{\cdot}6966.$

2. ·0024 × ·00012 = ·000000288. Here, as the product does not contain as many figures as are required, the deficiency is supplied by prefixing ciphers.

Rule. To divide decimals. Divide as in ordinary division, and then mark off, in the quotient, as many decimal places as are equal to the difference of the number in the dividend and divisor. The reason of this rule is apparent, as it is precisely the reverse of that of multiplication.

Thus 9·225 ÷ 2·5 = 3·69, where there are *three* decimal places in the dividend and *one* in the divisor; therefore we mark off *two* in the quotient.

When the number of places in the dividend is the same as in the divisor, the quotient will be a whole number and no point will be required. Thus 1·48 ÷ ·37 = 148 ÷ 37 = 4.

$$\text{Proof. } 1{\cdot}48 \div {\cdot}37 = \tfrac{148}{100} \div \tfrac{37}{100} = \tfrac{148}{100} \times \tfrac{100}{37} = \tfrac{148}{37} = 4.$$

When the number of decimal places in the dividend is less than the number in the divisor, make them equal by adding ciphers to the dividend (which will not alter its value), and then proceed as in the last case.

Thus 109·2 ÷ ·0012 = 109·2000 ÷ ·0012 = 1092000 ÷ 12 = 91000.

In order to carry on the divison, we must continue to' annex ciphers to the dividend until all the quotient figures are obtained, or a sufficient number of them. All the ciphers annexed must be counted as so many lamiced places. Thus to divide 3·2 by ·56 we have ;—

```
·56 ) 3·20000 ( 5·714, &c.
      280
      ———
       400
       392
       ———
        80
        56
        ——
       240
       &c.
```

Here, counting the ciphers annexed, we have *five* decimal places in the dividend and *two* in the divisor; therefore we mark off *three* in the quotient. By continuing to annex ciphers to the dividend, we may continue the process as far as we please, for in this example the quotient figures will not terminate.

Exercises.

(1.) 30·075 × ·16. (2.) ·06042 × 40. (3.) 128·09 × ·024. (4.) ·25 × ·04. (5.) 2·004 × ·25. (6.) ·016 × 2·125. (7.) 50·49 ÷ 2·2. (8.) 18·0023 ÷ ·087. (9.) 1 ÷ ·16. (10.) 41.296 ÷ 5·8. (11.) ·10324 ÷ ·29. (12.) ·0402 ÷ 134. (13.) 3·6 ÷ 8·4. (14.) ·02 × 2·5 × 3·4. (15.) ·05 × ·04 × 50. (16.) ·5 × ·05 × ·005.

Answers. (1.) 4·812. (2.) 2·4168. (3.) 3·07416. (4.) ·01. (5.) ·501. (6.) ·034. (7.) 22.95. (8.) 206·9. (9.) 6·25. (10.) 7·12. (11.) ·356. (12.) ·0003. (13.) ·428571, &c. (14.) ·17. (15.) ·1. (16.) ·000125.

31. *To convert a fraction into a decimal* it is only necessary to divide the numerator by the denominator, observing to annex ciphers to the numerator in order to continue the division. Thus to express $\frac{3}{8}$, $\frac{3}{40}$, $\frac{5}{16}$, in the form of a decimal, we have—

```
8 ) 3·000          40 ) 3·00          16 ) 5·0000
    —————               —————              ——————
    ·375 = 3/8.         ·075 = 3/40.       ·3125 = 5/16.
```

The operation, in the last example, is best performed by long division, or by dividing successively by 4 and 4. When the denominator of the fraction is an aliquot part of 10, 100, 1000, &c., the result is readily obtained in the following manner:—

$$\frac{3}{25} = \frac{3}{25} \times \frac{4}{4} = \frac{12}{100} = \cdot12\,;\ \frac{37}{125} = \frac{37}{125} \times \frac{8}{8} = \frac{296}{1000} = \cdot296.$$

Exercises. Reduce to decimals, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{32}$, $\frac{11}{80}$, $\frac{5}{64}$.

Answers. ·75, ·625, ·4375, ·09375, ·1375, ·078125.

In some cases the division will not stop, but the same figures will be repeated continually in the quotient. Decimals of this kind are called *Repeating* or *Circulating Decimals*, and the portion repeating is called the *Period* or *Repetend.*

1. Reduce $\frac{2}{3}$, $\frac{21}{90}$, $= \frac{2\cdot1}{9}$, $\frac{3}{110} = \frac{\cdot3}{11}$, to decimals.

```
3 ) 2·000          9 ) 2·1000          11 ) ·30000
    —————              ——————               ——————
    ·666, &c.          ·2333, &c.           ·02727, &c.
```

For the sake of conciseness, circulators are expressed by setting dots over the first and last figures of the repetend or period; thus, in the first example, the repeating figure is 6, and the circulator is expressed by $\cdot\dot{6}$; in the second example it is expressed by $\cdot2\dot{3}$; and in the third example by $\cdot0\dot{2}\dot{7}$.

When the period of the circulator begins immediately after the point it is called a *pure* circulator; all other circulators are called *mixed*. Thus $\cdot\dot{6}$ is a pure circulator, and $\cdot 2\dot{3}$ a mixed one.

Any fraction, in its least terms, containing 2 or 5, or any powers of them, as factors in the denominator, can always be reduced to a terminating decimal; but if the denominator contains any other factor besides those mentioned, then the corresponding decimal will not terminate, and the number of figures in the period can never *exceed* one less than the denominator.

Thus $\frac{1}{7} = \cdot\dot{1}4285\dot{7}$, where there are 6 figures in the period—that is, one less than 7; $\frac{1}{19}$ will be found to contain 18 figures in the period; but $\frac{1}{13} = \cdot\dot{0}7692\dot{3}$, where there are only 6 figures in the period.

32. *To reduce a circulator to a fraction.*

Rule 1. When the decimal is a pure circulator, write the period itself in the numerator, and place in the denominator as many 9's as there are figures in the period. Thus $\cdot\dot{2}\dot{7} = \frac{27}{99} = \frac{3}{11}$.

Proof. $\frac{1}{9} = \cdot 111$, &c., therefore multiplying by *any* number less than 9—say 4—we get $\frac{4}{9} = \cdot 444$, &c. $= \cdot\dot{4}$.

Again $\frac{1}{99} = \frac{1}{11} \times \frac{1}{9} = \frac{1}{11} \times \cdot 111$, &c. $= \cdot 010101$, &c., therefore multiplying by *any* number consisting of not more than two digits—say 23—we get $\frac{23}{99} = \cdot 232323$, &c. $= \cdot\dot{2}\dot{3}$.

In like manner $\frac{1}{999} = \frac{1}{111} \times \frac{1}{9} = \cdot 001001$, &c., therefore multiplying by *any* number consisting of not more than three digits—say 234—we get $\frac{234}{999} = \cdot 234234$, &c. $= \cdot\dot{2}3\dot{4}$. And so on.

Rule 2. When the decimal is a mixed circulator. Regarding only the figures after the point; for the numerator write down all the decimal figures to the end of the first period, subtracting the figures which do not circulate; for the denominator, write as many 9's as there are figures in the circulating part, followed by as many ciphers as there are figures in the part not circulating.

$$\text{Thus } \cdot 2\dot{3}\dot{4} = \frac{234 - 2}{990} = \frac{232}{990}$$

$$\textit{Proof.}\ \cdot 2\dot{3}\dot{4} = \tfrac{1}{10}(2 + \cdot\dot{3}\dot{4}) = \tfrac{1}{10}(2 + \tfrac{34}{99}) = \frac{2 \times 99 + 34}{990} =$$
$$\frac{2\,(100 - 1) + 34}{990} = \frac{234 - 2}{990} = \frac{232}{990}.$$

Exercises. Reduce to vulgar fractions $\cdot\dot{3}$, $\cdot\dot{6}$, $\cdot\dot{2}\dot{4}$, $\cdot\dot{3}6\dot{3}$, $\cdot 4\dot{9}$, $\cdot\dot{3}\dot{6}$, $\cdot 19\dot{4}$, $\cdot 0\dot{1}5\dot{3}$, $2\cdot 0\dot{6}$. *Ans.* $\frac{1}{3}$, $\frac{2}{3}$, $\frac{8}{33}$, $\frac{121}{333}$, $\frac{1}{2}$, $\frac{4}{11}$, $\frac{7}{36}$, $\frac{17}{1110}$, $2\frac{1}{15}$.

33. *Calculations with circulating decimals.*

In most cases a sufficient accuracy may be attained by repeating the periods several times.

1. Add together $3\cdot\dot{5}$, $2\cdot 0\dot{3}$, $\cdot\dot{6}\dot{7}$, $\dot{4}0\dot{6}$.

3·5555 2·0333 ·6767 ·4064 ——— 6·6720	Here, from the column not written down, we must obviously carry 1 to the next column. The sum is, therefore, true as far as the figures here written.

2. From $2\cdot0\dot{3}6\dot{2}$ take $\cdot\dot{6}3\dot{5}$, correct to 5 places.

2·03623 ·63563 ——— 1·40060	Here we do not carry anything from the figures not written down, therefore the subtraction is true as far as the figures here given.

In multiplication and division it is often best to reduce the circulators to fractions.

3. $\cdot3\dot{8} \times \cdot4\dot{9} = \frac{7}{18} \times \frac{1}{2} = \frac{7}{36} = \cdot19\dot{4}$.
4. $1\cdot0\dot{5} \times \cdot00\dot{2}\dot{7} = \frac{95}{90} \times \frac{3}{1100} = \frac{95}{33000} = \cdot002\dot{8}\dot{7}$.
5. $\cdot58\dot{3} \div \cdot0\dot{2}2\dot{7} = \frac{7}{12} \div \frac{1}{44} = \frac{77}{3} = 25\frac{2}{3}$ or $25\cdot\dot{6}$.
6. $5\cdot\dot{5} \div 2\cdot0\dot{5} = 5\frac{5}{9} \div 2\frac{5}{90} = \frac{50}{9} \times \frac{90}{185} = \frac{100}{37} = 2\cdot\dot{7}0\dot{2}$.
7. $\cdot\dot{2}\dot{7} \div \dot{3}\dot{6} = \frac{27}{99} \div \frac{36}{99} = \frac{27}{36} = \frac{3}{4}$.

Exercises. $\cdot\dot{5} \times \cdot\dot{3}$; $\cdot\dot{2}\dot{4} \times 3\cdot3$; $\cdot\dot{3}\dot{6} \times \cdot4\dot{9}$; $2\cdot0\dot{6} \times \cdot\dot{3}\dot{0}$; $\cdot\dot{6} \div \cdot\dot{3}$; $\cdot0\dot{1}5\dot{3} \div \cdot00\dot{1}\dot{7}$; $2\cdot\dot{1} \div \cdot\dot{3}\dot{8}$. *Ans.* $\cdot\dot{1}8\dot{5}$; $\cdot8$; $\cdot\dot{1}\dot{8}$; $\cdot\dot{6}\dot{2}$; 2; $8\cdot\dot{9}1\dot{8}$; $5\frac{1}{2}$.

34. *To find the value of a decimal of a given quantity.*

1. Find the value of 3·375 cwts.; and £5·3105.

3·375 cwts. 4 ——— 1·500 qrs. 28 ——— 14·0	£5·3105 20 ——— 6·2100 s. 12 ——— 2·52 d. 4 ——— 2·08 f.
Ans. 3 cwts. 1 qr. 14 lbs.	*Ans.* £5 6s. $2\frac{1}{2}$d.

In the first example, we multiply by 4 to bring the decimal part of cwts. to qrs., and then the decimal part of the qrs. by 28 to bring them to lbs., and so on.

2. Find the value of $£2\cdot\dot{4}\dot{5}$; and $3\cdot\dot{6}\dot{3}$ of a day.

$$£2\cdot\dot{4}\dot{5} = £2\tfrac{45}{99} = £2\tfrac{5}{11} = £2 \text{ 9s. } 1\tfrac{1}{11}\text{d.}$$

$$3\,\dot{6}\dot{3} = 3\tfrac{63}{99} = 3\tfrac{7}{11} \text{ days} = 3 \text{ d. } 15 \text{ h. } 16 \text{ min. } 21\tfrac{9}{11} \text{ sec.}$$

Exercises. Find the value of £2·75: £·65; 3·25s.; 2·75d.; 3·55 guineas; $£1\cdot\dot{3}\dot{6}$; 2·35 tons; ·25 yds.; 2·15 days; 1·55 crowns; 1·45 acres; $£\cdot0\dot{6}$; $2\cdot4\dot{9}$ ft. *Ans.* £2 15s.: 13s.; 3s. 3d.; $2\frac{3}{4}$d.; £3 14s. $6\frac{2}{3}$d.; £1 7s. $3\frac{3}{11}$d.; 2 tons 7 cwt.; 9 in.; 2 d. 3 h. 36 min.; 7s. 9d.; 1 ac. 1 r. 32 p.; 1s. 4d.; 2 ft. 6 in.

35. *To reduce a given quantity to the decimal of another.*

1. Reduce 5s. $4\frac{1}{2}$d. to the decimal of a pound.

$$\begin{array}{r|l} 4 & 2\cdot0 \\ 12 & 4\cdot500 \\ 20 & 5\cdot3750 \\ & £\cdot26875 \text{ Ans.} \end{array}$$

Here we bring the two farthings to the decimal of a penny by dividing by 4, which gives ·5d.; then prefixing the 4d., we bring 4·5d. to the decimal of a shilling by dividing by 12; and so on.

Or thus. 5s. $4\frac{1}{2}$d. $= 64\frac{1}{2}$d. $= \frac{129}{2}$ d. $= \frac{129}{2 \times 12 \times 20}$ £ $= \frac{43}{160}$ £ $=$ £·26875.

2. Reduce 5 yds. 2 ft. $7\frac{4}{5}$ in. to the decimal of a yard.

$$\begin{array}{l} 12\,)\,7\cdot80 = 7\frac{4}{5} \\ \overline{} \\ 3\,)\,2.650 \\ \overline{} \\ 5\cdot88\dot{3} \text{ yds.} \end{array}$$

Here we bring 7·8 in. to the decimal of a ft. by dividing by 12, which gives ·65 feet; then prefixing the 2 feet we have 2·65 feet to bring to yards, which we do by dividing by 3, and so on.

3. Reduce 2s. 6d. to the decimal of 10s. 5d.; 7s. 6d. to the decimal of £5.

$$\frac{2s.\ 6d.}{10s.\ 5d.} = \frac{30d.}{125d.} = \frac{6}{25} = \cdot24.$$

$$7s.\ 6d. \div £5 = 7\tfrac{1}{2}s. \div 500s. = 7\cdot5 \div 500 = \cdot015.$$

When a concrete quantity is to be multiplied or divided by a decimal, the general rule is, first bring the quantity to a decimal of the highest unit in it, and then multiply or divide, as the case may be; but it is often most convenient to reduce the concrete quantity to its lowest denomination, and then to multiply or divide; as shown in the following examples:—

4. Multiply and divide £9 8s. 3d. by 6·74.

£9 8s. 3d. = £9·4125; also £9 8s. 3d. = 2259d.

First, £9·4125 × 6·74 = £63·44025 = £63 8s. $9\frac{1}{2}$d.
2259d. × 6·74 = 15225·66d. = £63 8s. $9\frac{1}{2}$d.

Second, £9·4125 ÷ 6·74 = £1·3965 = £1 7s. 11d.
2259d. ÷ 6·74 = 335·16d. = £1 7s. 11d.

Exercises. Reduce

1. 2s. $3\frac{1}{2}$d.; $3\frac{3}{4}$d.; 15s. $2\frac{1}{4}$d.: 3 guineas; $16\frac{1}{5}$d.; $6\frac{1}{4}$s.; to the decimal of £1. *Ans.* $\cdot11458\dot{3}$; ·015625; ·759375; 3·15; ·0675; ·3125.

2. 3d.; $7\frac{1}{2}$d.; $\frac{3}{4}$d.; 2s. $1\frac{4}{5}$d.; $5\frac{1}{2}$d.; $3\frac{3}{5}$d.; to the decimal of a shilling. *Ans.* ·25; ·625: ·0625; 2·15; $\cdot458\dot{3}$; ·3.

3. 7 lbs.; 1 qr. $3\frac{1}{2}$ lbs.; 2 qrs. 1 lb. 12 oz.; $22\frac{2}{5}$ lbs.; $1\frac{3}{4}$ lbs.; to the decimal of a cwt. *Ans.* ·0625; ·28125; ·515625; ·2; ·015625.

4. 1 qr. 2 nails; 2 ft. $6\frac{3}{5}$ in.; 2 ft. 3 in.; $3\frac{3}{5}$ in.; 2 yds. 1 nail; to the decimal of a yard. *Ans.* ·375; ·85; ·75; ·1; 2·0625.

5. 28 lbs.; 1 cwt. 56 lbs.; 1 qr. 7 lbs.; 1 cwt. 14 lbs.; to the decimal of 4 cwts. *Ans.* ·0625; ·375; ·078125; ·28125.

36. *Problems in decimals.*

1. Express the sum and difference of ·245 and ·205 in the form of a fraction.

2. Reduce $\frac{70}{125}$, $\frac{2}{50}$, $\frac{3}{20}$, and $\frac{2}{5}$ of $1\frac{3}{8}$ of 3·13, to decimals; and the difference of 5·274 and 5·024 to a fraction.

3. Reduce $\frac{3}{44}$ and $5\frac{3}{28}$ to decimals; ·075 and $\cdot\dot{6}\dot{3}$ to fractions.

4. Find the value of $2\frac{3}{8} + 5\frac{3}{5} + 6\frac{1}{2} - 12\frac{7}{40} + 7\frac{3}{40}$, both by vulgar fractions and by decimals, and show that the two results coincide.

5. Multiply 2s. 8d. by 3·23, and divide the result by ·8.

```
      3·23
       32 d. = 2s. 8d.
      ----
       646
      969
      ----
·8 ) 103·36d.
      ----
      129·2d. = 10s. 9⅕d.
```

Or thus.

$$\frac{32\text{d.} \times 3{\cdot}23}{{\cdot}8} = 40\text{d.} \times 3{\cdot}23 = 129{\cdot}2\text{d.} — \text{\&c.}$$

6. Multiply £5 4s. $3\frac{1}{2}$d. by 4·02, and add £2·316 to the result.
7. Find the sum of 6s. 7d. ÷ 2·42, and 3s. $3\frac{1}{2}$d. ÷ 1·21.
8. Multiply 1·6375 by 6·4; and divide ·125 by ·005.
9. Multiply 12s. $1\frac{1}{2}$d. by 85·3125; and divide £42 5s. by 29·25.
10. Multiply 3 cwt. 6 lbs. by 5·125; and divide £19507 9s. $4\frac{1}{2}$d. by 234·5.
11. What is the rent of 24·04 acres of land at £2·25 per acre?
12. What decimal multiplied by ·8 will give the sum of $\frac{1}{5}$, $\frac{2}{13}$, $\frac{2}{3}$?
13. Add together $4\frac{1}{3}$, $2\frac{1}{5}$, $6\frac{1}{4}$, $2\frac{7}{8}$, 23.568, $3\cdot\dot{6}$, ·005.
14. Reduce 1s. $8\frac{1}{4}$d. to the fraction and to the decimal of £1.
15. Find the value of $\cdot\dot{3}7\dot{5} + \cdot\dot{6}2\dot{4}$; $\cdot\dot{2} + \cdot\dot{3}$; $\cdot\dot{4} \div \cdot\dot{2}$; $\cdot\dot{2}\dot{4} \div \cdot\dot{3}\dot{2} - \cdot\dot{2}\dot{7} \div \cdot\dot{3}\dot{6}$; $\cdot\dot{3}6\dot{3} \div \cdot\dot{2}4\dot{2}$; $(\cdot6 - \frac{3}{7}) \div \cdot01$; $(\cdot\dot{2}\dot{7} - \frac{2}{11}) \times \cdot0011$.
16. Find the value of £18 9s. $4\frac{1}{2}$d. × ·0184 ÷ ·023.
17. Add together £$2\cdot\dot{3}\dot{6}$, £$1\cdot\dot{5}\dot{4}$, 1·125s. and 6·75d.
18. 40·85 at £2 5s. $1\frac{5}{8}$d.; 259·8 at £8·64; 163·7 at £·34.
19. Find the fraction equivalent to ·75 + ·25 — ·45 + ·15.
20. Find the value of 1·6375 of £5; and 2·175 of £20.
21. Find the value of 5s. 4d. × 3·48 + 16d. × 1·74 — 4s. × ·87.
22. $\frac{4}{5}$ of 1·3125 guineas; £$\cdot\dot{2}\dot{7} \times \cdot22$; £$\cdot\dot{6}\dot{3} \div \cdot\dot{1}\dot{8}$.
23. $2\cdot248 \times 4\cdot68 \times 4\cdot\dot{4} \times 5 \div 1\cdot56 \times \cdot562 \times 8\cdot\dot{8} =$

Answers. (1.) $\frac{9}{20}$, $\frac{1}{25}$. (2.) ·56, ·04, ·15, 1·7215; $\frac{1}{4}$. (3.) $\cdot06\dot{8}\dot{1}$; $5\cdot10\dot{7}14285\dot{}$; $\frac{3}{40}$; $\frac{7}{11}$. (4.) $9\frac{3}{8}$ = 9·375. (6.) £23 5s. $6\frac{3}{4}$d. (7.) 5s. $5\frac{1}{4}$d. (8.) 10·48; 25. (9.) £51 14s. $4\frac{3}{4}$d.; £1 8s. $10\frac{2}{3}$d. (10.) 15 cwt. 2 qrs. $16\frac{3}{4}$ lbs.; £83 3s. 9d. (11.) £54 1s. $9\frac{3}{5}$d. (12.) ·125. (13.) 42·8979. (14.) $\frac{27}{320}$; ·084375. (15.) 1; $\frac{5}{9}$; 2; 0; $1\frac{1}{2}$; $17\frac{1}{7}$; ·0001. (16.) £14 15s. 6d. (17.) £3 19s. $10\frac{19}{44}$d. (18.) £92 3s. $9\frac{1}{4}$d.; £2244 13s. $5\frac{1}{4}$d.; £55 13s. $1\frac{23}{25}$d. (19.) $\frac{7}{10}$. (20.) £8 3s. 9d.; £43 10s. (21.) 17s. $4\frac{1}{5}$d. (22.) £1 2s. $0\frac{3}{5}$d.; 1s. $2\frac{2}{5}$d.; £3 10s. (23.) 30.

PRACTICE.

37. Practice is an expeditious method of finding the value of any quantity of merchandise, the value of an unit being given. The following aliquot parts will be readily remembered:—

10s. = $\frac{1}{2}$ £	6s. 8d. = $\frac{1}{3}$ £	6d. = $\frac{1}{2}$s.	4d. = $\frac{1}{3}$s.
5s. = $\frac{1}{4}$ £	3s. 4d. = $\frac{1}{6}$ £	3d. = $\frac{1}{4}$s.	2d. = $\frac{1}{6}$s.
2s. 6d. = $\frac{1}{8}$ £	1s. 8d. = $\frac{1}{12}$ £	$1\frac{1}{2}$d = $\frac{1}{8}$s.	1d. = $\frac{1}{12}$s.

1. Find the price of 214 articles at £3 each.
   Value 214 at £1 = £214.
   ∴ Value 214 at £3 = 3 times £214 = £642.

2. Required the value of 31 lbs. of tea at 7s. per lb.
   Value 31 at 1s. = 31s.
   ∴ Value 31 at 7s. = 7 times 31s. = 217s. = £10 17s.

3. Find the value of 117 at £5; 57 at £8; 41 at 3s.; 17 at 6s.
   Answers. £585; £456; £6 3s.; £5 2s.

4. Find the value of 321 articles at 5s. each.
   Here 5s. is the fourth of £1.
   321 at £1 = £321.
   321 at 5s. = $\frac{1}{4}$ of £321 = £80 5s.

$$\begin{array}{r|r} & £321 \\ \hline 5s. = \frac{1}{4} & 80\ \ 5 \end{array}$$

5. Required the value of 672 yards of cloth at 6s. 8d. per yard.
   Here 6s. 8d. is the third of £1.
   672 at £1 = £672.
   672 at 6s. 8d. = $\frac{1}{3}$ of £672 = £224.

$$\begin{array}{r|r} & £672 \\ \hline 6s.\ 8d. = \frac{1}{3} & 224 \end{array}$$

6. 274 at 4s.; 375 at 3s. 4d. *Answers.* £54 16s.; £62 10s.

7. Find the value of 123 at £4 5s. each.
   Here 5s. is the fourth of £1.

$$\begin{array}{rr} & £ \\ 123 \text{ at } £1 = & 123 \\ & 4 \\ \hline 123 \text{ at } £4 = 4 \text{ times } £123 = & 492 \\ 123 \text{ at } 5s. = \text{the 4th of } £123 = & 30\ \ 15 \\ \hline \therefore 123 \text{ at } £4\ 5s. = & 522\ \ 15 \end{array}$$

$$\begin{array}{r|r} & 123 \\ & 4 \\ \hline & 492 \\ 5s. = \frac{1}{4} & 30\ \ 15 \\ \hline & £522\ \ 15 \end{array}$$

8. 217 at £3 10s.; 36 at £2 4s. *Answers.* £759 10s.; £79 4s.

9. Required the value of 248 at 7s. 6d.
   7s. 6d. = 5s. + 2s. 6d.; 5s. is the $\frac{1}{4}$th. of £1; 2s. 6d. the $\frac{1}{8}$th of £1.

$$\begin{array}{lr} 248 \text{ at } £1 \quad = £248. & \\ 248 \text{ at } 5s. \quad = \frac{1}{4} \text{ of } £248 = & £62 \\ 248 \text{ at } 2s.\ 6d. = \frac{1}{8} \text{ of } £248 = & 31 \\ \hline \therefore 248 \text{ at } 7s.\ 6d. = & £93 \end{array}$$

$$\begin{array}{r|r} & 248 \\ \hline 5s. = \frac{1}{4} & 62 \\ 2s.\ 6d. = \frac{1}{8} & 31 \\ \hline & £93 \end{array}$$

Or thus. In the foregoing solution both portions of the price are taken as aliquot parts of £1; but in the following solution one portion is taken as an aliquot part of the other—that is, the value of 2s. 6d. is taken as the half of the value of 5s.

$$\begin{array}{lr} 248 \text{ at } 5s. \quad = \frac{1}{4} \text{ of } £248 = & £62 \\ 248 \text{ at } 2s.\ 6d. = \frac{1}{2} \text{ of } £62 \quad = & £31 \\ \hline \text{at } 7s.\ 6d. = & £93 \end{array}$$

$$\begin{array}{r|r} & 248 \\ \hline 5s. = \frac{1}{4} & 62 \\ 2s.\ 6d. = \frac{1}{2} & 31 \\ \hline & £93 \end{array}$$

Or thus. First multiply the number of articles by the 7s., and taking 6d. as the half of 1s., we have

248 at 1s.	=	248s.
		7
248 at 7s. = 7 times 248s.	=	1736
248 at 6d. = $\frac{1}{2}$ of 248s.	=	124
	2,0)	186,0
248 at 7s. 6d.	=	£93

	248s.
	7
	1736
6d. = $\frac{1}{2}$	124
2,0)	186,0
	£93

10. 213 at 12s. 6d.; 37 at 2s. 4d. *Answers.* £133 2s. 6d.; £4 6s. 4d.

11. Required the value of 26 at 6s. 9d. each.

Here after multiplying by the shillings, the remainder of the price 9d. = 6d. + 3d.; then 6d. is the half of 1s., and 3d. is the half of 6d.

26 at 1s.	=	26s.
		6
26 at 6s. = 6 times 26s.	=	156
26 at 6d. = $\frac{1}{2}$ of 26s.	=	13
26 at 3d. = $\frac{1}{2}$ of 13s.	=	6 6
	2,0)	17,5 6
∴ 26 at 6s. 9d.	=	£8 15 6

	26s.
	6
	156
6d. = $\frac{1}{2}$	13
3d. = $\frac{1}{2}$	6 6
2,0)	17,5 6
	£8 15 6

Or thus. This question may be solved more concisely by taking 9d. as the aliquot part of 6s.; the 9d. being the eighth of 6s.

26 at 6s. = 6 times 26s.	=	156s.
26 at 9d. = $\frac{1}{8}$th of 156s.	=	19 6
	2,0)	17,5 6
		£8 15 6

	26
	6
	156
9d. = $\frac{1}{8}$	19 6
2,0)	17,5 6
	£8 15 6

When the price consists of shillings and pence only, it is generally best to multiply the number of articles by the shillings, and take the aliquot parts for the pence, as in the foregoing solutions.

12. 412 at 5s. 10d.; 702 at 3s. 9d.; 513 at 8s. 8d.; 36 at 5s. 5d.
Answers. £120 3s. 4d.; £131 12s. 6d.; £222 6s.; £9 15s.

13. Find the price of 87 cwt. of coffee at £5 7s. 4d. per cwt.

Here we take 7s. 4d. = 4s. + 3s. 4d.; 4s. is the fifth of £1; and 3s. 4d. is the sixth of £1.

87 at £1	=	£87
		5
87 at £5 = 5 times £87	=	435
87 at 4s. = $\frac{1}{5}$ of £87	=	17 8
87 at 3s. 4d. = $\frac{1}{6}$ of £87	=	14 10
∴ 87 at £5 7s. 4d.	=	£466 18

	£87
	5
	435
4s. = $\frac{1}{5}$	17 8
3s. 4d. = $\frac{1}{6}$	14 10
	£466 18

Or thus. 7s. 4d. = 6s. 8d. + 8d.; 6s. 8d. is the 3rd of £1; and 8d. is the 10th of 6s. 8d.

87 at £1	=	£87
		5
87 at £5 = 5 times £87	=	435
87 at 6s. 8d. = $\frac{1}{3}$ of £87	=	29
87 at 8d. = $\frac{1}{10}$ of £29	=	2 18
		£466 18

	£87
	5
	435
6s. 8d. = $\frac{1}{3}$	29
8d. = $\frac{1}{10}$	2 18
	£466 18

14. 221 at £3 12s. 6d.; 357 at £2 8s. 4d.; 116 at £6 3s. 8d.
Answers. £801 2s. 6d.; £862 15s.; £717 5s. 4d.

15. Find the value of 260 at £2 11s. 7½d.

11s. 7½d. = 10s. + 1s. + 6d. + 1½d., where 1s. is the 10th of 10s., 6d. the half of 1s., and 1½d. the 4th of 6d.

260 at £1	=			£260
				2
260 at £2	= 2 times £260	=		520
260 at 10s.	= ½ of £260	=		130
260 at 1s.	= $\frac{1}{10}$ of £130	=		13
260 at 6d.	= ½ of £13	=		6 10
260 at 1½d.	= ¼ of £6 10s.	=		1 12 6
260 at £2 11s. 7½d.		=		£671 2 6

	£260
	2
	520
10s. = ½	130
1s. = $\frac{1}{10}$	13
6d. = ½	6 10
1½d. = ¼	1 12 6
	£671 2 6

The aliquot parts should, if possible, be taken, so that none of the divisors shall exceed 12.

16. 32 at £3 11s. 8½d.; 27 at £3 16s. 8d.; 36 at £10 10s. 10d.
Answers. £114 14s. 8d.; £103 10s.; £379 10s.

17. Find the value of 213 at 10d.; 42 at 7s. 2½d.

10d. = 6d. + 4d.; 6d. = ½ of 1s.; 4 = ⅓ of 1s.

213 at 1s.	=	213s.
213 at 6d.	= ½ of 213s. =	106 6
213 at 4d.	= ⅓ of 213s. =	71
	2,0)	17,7 6
213 at 10d.	=	£8 17 6

	213
6d. = ½	106 6
4d. = ⅓	71 0
2,0)	17,7 6
	£8 17 6

Again, after finding the value at 7s., the remainder of the price, 2½d. = 2d. + ½d.; then 2d. = ⅙ of 1s.; and ½d. = ¼ of 2d.

42 at 1s.	=	42s.
		7
42 at 7s.	= 7 times 42s. =	294
42 at 2d.	= ⅙ of 42s. =	7
42 at ½d.	= ¼ of 7s. =	1 9
	2,0)	30,2 9
42 at 7s. 2½d.	=	£15 2 9

	42
	7
	294
2d. = ⅙	7
½d. = ¼	1 9
2,0)	30,2 9
	£15 2 9

18. 32 at 8d.; 231 at 4d.; 371 at 3d.; 36 at 7d.; 306 at 19s. 6½d.
Answers. £1 1s. 4d.; £3 17s.; £4 12s. 9d.; £1 1s.; £298 19s. 9d.

When the price is a whole number of pounds, or a whole number of shillings, less by some aliquot part of a pound or a shilling, as the case may be, the calculation is generally very much simplified by calculating for the whole number and then subtracting from the result the value of the price added to the true price, as in the following examples;—

19. Find the value of 75 at £4 17s. 6d.; 96 at 7s. 11d.

Here the price, in the first example, is 2s. 6d. less than £5, and 2s. 6d. = ⅛ of £1.

75 at £1	=	£75
		5
75 at £5	= 5 times £75 =	375
75 at 2s. 6d.	= ⅛ of £75 =	9 7 6
∴ 75 at £4 17s. 6d.	=	£365 12 6

	£75
	5
	375
2s. 6d. = ⅛	9 7 6
	£365 12 6

Again, the price, in the second example, is 1d. less than 8s., and 1d. = $\frac{1}{12}$ of 1s.

96 at 1s. =	96s.			96
	8			8
96 at 8s. = 8 times 96s. =	768			768
96 at 1d. = $\frac{1}{12}$ of 96s. =	8		1d. = $\frac{1}{12}$	8
2,0)	76,0		2,0)	76,0
96 at 7s. 11d. =	£38			£38

20. 649 at £3 18s. 4d.; 181 at £5 16s. 8d.; 23 at 6s. 10d.; 57 at 9s. 10½d.; 84 at 4s. 10½d. *Answers.* £2541 18s. 4d.; £1055 16s. 8d.; £7 17s. 2d.; £28 2s. 10½d.; £20 9s. 6d.

When the price is nearly a whole number of pounds or shillings, it will be often most convenient to calculate for the whole number, and then subtract from the result the value of the part added to the true price; as in the following examples:—

21. 83 at £4 9s. 11¾d.; 98 at 2s. 11d.

Here the price only wants ¼d. to make £4 10s.

83 at £1 =	£83		
	4		
83 at £4 = 4 times £83 =	332		
83 at 10s. = ½ of £83 =	41	10	
83 at £4 10 =	373	10	
83 at ¼d. =		1	8¾
83 at £4 9s. 11¾d. =	£373	8	3¼

In the second example, the price only wants 1d. to make 3s.; then 98 at 2s. 11d. = 98 × 3s. — 98 × 1d. = 294s. — 8s. 2d. = £14 5s. 10d.

22. 105 at £9 19s. 11d.; 92 at £7 9s. 11¾d.; 48 at £4 3s. 10d.; 72 at 4s. 11d.; 846 at 7s. 11¾d. *Answers.* £1049 11s. 3d.; £689 18s. 1d.; £201 4s.; £17 14s.; £337 10s. 4½d.

Sometimes the calculation is simplified by taking the same aliquot part twice; as in the following examples:—

23. 221 at £3 13s. 4d.; 291 at 8d.

Here 13s. 4d. = 6s. 8d. + 6s. 8d.; and 6s. 8d. = ⅓ of £1.

221 at £1 =	£221		
	3		
221 at £3 =	663		
221 at 6s. 8d. = ⅓ of £221 =	72	13	4
221 at 6s. 8d. = ,, =	73	13	4
221 at £3 13s. 4d. =	£810	6	8

In the second example, 8d. = 4d. + 4d.; and 4d. = $\frac{1}{3}$ 1s.

	291 at 1s.	= 291s.
291 at 4d. = $\frac{1}{3}$ of 291s.	=	97
291 at 4d. = ,,	=	97
	291 at 8d.	= 194 = £9 14s.

24. 200 at £4 13s. 4d.; 892 at 4s. 8d. *Ans.* £933 6s. 8d.; £208 2s. 8d.

Sometimes the shillings and pence in the price may be taken as an aliquot part of the pounds in the price; and similarly, the pence and farthings in the price may be taken as an aliquot part of the shillings in the price. In such cases the calculation may be often much simplified; as in the following examples:—

25. 225 at £5 12s. 6d.; 122 at 3s. 9d.

Here 12s. 6d. = $\frac{1}{8}$ of £5. And in the second example 9d. = $\frac{1}{4}$ of 3s.

	£225				122s.	
	5				3	
	1125				366	
12s. 6d. = $\frac{1}{8}$	140 12 6			9d. = $\frac{1}{4}$	91 6	
	£1265 12 6			2,0)	45,7 6	
					£22 17 6	

26. 321 at £4 16s.; 23 at 5s. 7½d.; 133 at 2s. 8d.; 40 at £3 15s.
Answers. £1540 16s.; £6 9s. 4½d.; £17 14s. 8d.; £150.

In the following examples the quantity contains a fraction:—

27. Find the value of 192⅔ at £5 12s. 6d. each.

Here we first find the value of the whole number, 192, as in the foregoing examples, and then add the value of ⅔ of the price of one article.

192 at £1	=	£192
		5
192 at £5	=	960
192 at 10s. = $\frac{1}{2}$ of 192	=	96
192 at 2s. 6d. = $\frac{1}{4}$ of £96	=	24
192 at £5 12s. 6d.	=	1080
⅔ of £5 12s. 6d.	=	3 15
∴ 192⅔ at £5 12s. 6d.	=	£1083 15

	192
	5
	960
10s. = $\frac{1}{2}$	96
2s. 6d. = $\frac{1}{4}$	24
For ⅔	3 15
	£1083 15

Where the value of ⅔ is found by multiplying £5 12s. 6d. by 2 and dividing by 3.

28. 382⅔ at £2 9s. 6d.; 247⅔ at £2 16s. 8d.; 139⅓ at 16s. 8d.; 117½ at 12s. 4d.; 495¼ at £4 12s. 6d. *Answers.* £946; £700 19s. 4d.; £116 2s. 11d.; £72 9s. 2d.; £2290 6s.

When the quantity only wants one fractional part to make it an integer, it is often best to calculate for the value of this integer, and subtract the value of the fractional part which has been added.

29. 281⅞ at £3 15s.; 572¾ at 7s. 3½d.

	£282				573	
	3				7	
	846				4011	
10s. = ½	141			3d. = $\frac{1}{28}$	143	3
5s. = ½	70	10		½ = ⅙	23	10½
For 282	1057	10		For 573	4178	1½
For ⅛		9 4½		For ¼	1	9⅞
For 281⅞	£1057	0 7½			2,0) 417,6	3⅝
				For 572¾	£208 16	3⅝

In the first example, we calculate for the value of 282, but as this is ⅛ too much, we subtract from it the value of ⅛.

30. 29⅝ at £2 5s. 4½d.; 69¾ at 8s. 4d. *Ans*. £67 16s. 2½d.; £29 1s. 3d.

In the following examples, all the terms of the quantity are not in the same denomination :—

31. Required the value of 7 lbs. 10 oz. of tea at 2s. 6d. per lb.

	s.	d.
Value 1 lb. =	2	6
		7
Value 7 lbs. =	17	6
Value 8 oz. = ½ of value 1 lb. =	1	3
Value 2 oz. = ¼ of value 8 oz. =		7½
∴ Value 7 lbs. 10 oz. =	19	4½

32. Find the value of 2 cwt. 3 qrs. 16 lbs. at £2 16s. 6d. per cwt.

	2 16 6	=	Value 1 cwt.		
	2				
	5 13 0	=	,,	2 cwt.	
2 qrs. = ½	1 8 3	=	,,	2 qrs. or ½ of 1 cwt.	
1 qr. = ½	14 1½	=	,,	1 qr. or ½ of 2 qrs.	
14 lbs. = ½	7 0¾	=	,,	14 lbs. or ½ of 1 qr.	
2 lbs. = $\frac{1}{7}$	1 0	=	,,	2 lbs. or $\frac{1}{7}$ of 14 lbs.	
	£9 3 5¼	=	,,	2 cwt. 3 qrs. 16 lbs.	

33. Find the value of 122 cwt. 2 qrs. 11½ lbs. at £5 12s. 6d. per qr.

122 cwt. 2 qrs. 11½ lbs. = 490 qrs. 11½ lbs.

	£		
	490		
	5		
	2450		
10s. = ½	245		
2s. 6d. = ¼	61	5	
For 11½ lbs.	2	6	$2\frac{13}{28}$
	£2758	11	$2\frac{13}{28}$

	£	s.	d.
Value 1 qr. =	5	12	6
Value 7 lbs. = ¼ of 1 qr. =	1	8	1½
,, 4 lbs. = $\frac{1}{7}$ of 1 qr. =		16	$0\frac{6}{7}$
,, ½ lb. = ⅛ of 4 lbs. =		2	$0\frac{3}{28}$
∴ Value 11½ lbs. =	£2	6	$2\frac{13}{28}$

Here we first calculate, in the usual manner, the value for 490 qrs. and to this we add the value for 11½ lbs.

34. 5 lbs. 8 oz. at 6s. 8d. per lb.; 3 cwt. 2 qrs. 8 lbs. at £3 4s. 8d. per cwt.; 75 cwt. 3 qrs. 12 lbs. at 11s. 8d. per qr.; 34 cwt. 1 qr. 3$\frac{1}{2}$ lbs. at £1 3s. 4d. per qr.

Answers. £1 16s. 8d.; £11 10s. 11$\frac{3}{7}$d.; £177; £159 19s. 7d.

When the quantity is equal to an integral number of the units, whose price is given, less by some aliquot part of a unit, the calculation may be simplified by calculating for the value of the integral number, and then subtracting from the result the value of the part which this integral number is in excess of the proposed quantity; as in the following examples:—

35. Find the value of 11 qrs. 21 lbs. at 16s. 8d. per qr.

Here the quantity wants 7 lbs. to make it 12 qrs., and 7 lbs. = $\frac{1}{4}$ of 1 qr. Hence we proceed as follows:—

		s.	d.		
		16	8	=	Value 1 qr.
			12		
	10	0	0	=	„ 12 qrs.
7 lbs. = $\frac{1}{4}$		4	2	=	„ 7 lbs. or $\frac{1}{4}$ of 1 qr.
	£9	15	10	=	„ 11 qrs. 21 lbs.

36. 3 qrs. 24 lbs. at 14s. 7d. per qr.; 1 qr. 1 lb. 12 oz. at 5s. 10d. per lb. *Answers.* £2 16s. 3d.; £8 13s. 6$\frac{1}{2}$d.

When the quantity is nearly an integral number of the units, whose price is given, it will be often most convenient to calculate for this integral, and then subtract from the result the value of the part which this integral number is in excess of the proposed quantity; as in the following examples:—

37. Find the value of 7 lbs. 15$\frac{3}{4}$ oz. at £1 4s. 8d. per lb.

£	s.	d.		
1	4	8		
		8		
9	17	4	=	Value 8 lbs.
		4$\frac{5}{8}$	=	„ $\frac{1}{4}$ oz.
£9	16	11$\frac{3}{8}$	=	„ 7 lbs. 15$\frac{3}{4}$ oz.

Here the price of the lb. divided by 16, or by 4 and 4, will give the price of the oz., and this result divided by 4 will give that of the $\frac{1}{4}$ oz.

38. 3 cwt. 2 qrs. 27$\frac{1}{3}$ lbs. at £2 16s. per qr.; 1 cwt. 3 qrs. 16 lbs. 15$\frac{2}{3}$ oz., at 5s. 4d. per lb. *Answers.* £41 19s. 7$\frac{1}{5}$d.; £56 15s. 10$\frac{2}{3}$d.

TARE AND TRET.

38. The *gross weight* is the weight of both goods and packages.

Tare is an allowance made to the buyer for the weight of the package, either at so much per box, barrel, &c., or at so much per cwt. In the

former case the Tare is found by multiplication; in the latter case by practice.

Tret is an allowance of 1 lb. in 26 lbs., or $\frac{1}{26}$ of the weight, given on goods liable to waste.

Cloff is an allowance (after Tare and Tret have been deducted) of 2 lbs. on 3 cwt., given to retailers for turning the scales.

Suttle is the name given to the remainder, after any of the foregoing allowance has been made.

Net weight is what remains after all allowances have been deducted.

1. What is the net weight of 6 chests of tea, each 3 qrs. 8 lbs., tare at 22 lbs. per chest?

	cwt.	qr.	lbs.	
	0	3	8	
			6	
	4	3	20	gross
22 lbs. × 6 =	1	0	20	tare
	3	3	0	net

2. What is the net weight of 42 hogsheads, each 64 cwt. 3 qrs. 10 lbs., tare at 74 lbs. per hogshead? *Ans.* 2695 cwt. 2 qrs.

3. What is the net weight of 59 casks of butter weighing 42 cwt. 1 qr. 9 lbs., tare at 12 lbs. per cask? *Ans.* 36 cwt. 0 qr. 1 lb.

4. What is the net weight of 168 cwt. 1 qr. 21 lbs., tare 21 lbs. per cwt.?

	cwt.	qr.	lbs.	
	168	1	21	gross
14 lbs. = $\frac{1}{8}$	21	0	$6\frac{1}{8}$	
7 lbs. = $\frac{1}{2}$	10	2	$3\frac{1}{16}$	
	31	2	$9\frac{3}{16}$	tare
	136	3	$11\frac{13}{16}$	net

5. Required the net weight of 1573 cwt. 3 qrs. 14 lbs., tare 24 lbs. per cwt. *Ans.* 1236 cwt. 2 qrs. 13 lbs.

6. Required the net weight of 18 bags of raisins, each 17 cwt. 0 qr. 10 lbs., tare 14 lbs. per cwt. *Ans.* 269 cwt. 0 qr. $17\frac{1}{2}$ lbs.

7. Required the net weight of 57 cwt. 1 qr., tare 12 lbs. per cwt.

	cwt.	qr.	lbs.	
	57	1	0	gross
			3	
28)	171	3	0	
12 lbs. = $\frac{3}{28}$ of 1 cwt.	6	0	15	tare
	51	0	13	net

Or thus. Tare for 57 cwt. = 12 lbs. × 57 = 684 lbs.
 ,, 1 qr. = $\frac{1}{4}$ of 12 lbs. = 3

	cwt.	qr.	lbs.
Tare 687 =	6	0	15
Gross	57	1	0
Net	51	0	13

8. Required the net weight of 16 cwt. 1 qr. 4 lbs., tare 17$\frac{1}{2}$ lbs. per cwt. *Ans.* 13 cwt. 2 qrs. 27 lbs.

9. Required the net weight of 16 cwt. 2 qrs., tare 16 lbs. per cwt. and tret as usual.

	cwt.	qrs.	lbs.
	16	2	0 gross
16 lbs. = $\frac{1}{7}$	2	1	12 tare
26)	14	0	16 suttle
		2	4$\frac{12}{13}$ tret
	13	2	11$\frac{1}{13}$ net

10. Required the net weight of 6 cwt. 1 qr. 14 lbs., tare 14 lbs. per cwt., and tret as usual. *Ans.* 5 cwt. 1 qr. 12 lbs.

11. Required the net weight of 2 cwt. 1 qr. 14 lbs., tare 8 lbs. per cwt., and tret as usual. *Ans.* 2 cwt. 0 qr. 13$\frac{1}{2}$ lbs.

12. Required the net weight of 31 cwt. 3 qrs. 12 lbs., tare 7 lbs. per cwt., tret and cloff as usual.

	cwt.	qrs.	lbs.
	31	3	12 gross
7 lbs. = $\frac{1}{16}$	1	3	27 tare
26)	29	3	13 suttle
	1	0	16 tret
168)	28	2	25 suttle
2 lbs. = $\frac{1}{168}$ of 3 cwt.			19 cloff
	28	2	6 net

13. Required the net weight of 79 cwt. 3 qrs. 14 lbs., tare 18 lbs. per cwt., tret and cloff as usual. *Ans.* 64 cwt. 0 qr. 8 lbs.

14. Required the net weight of 7 hogsheads of tobacco, each 11 cwt. 0 qr. 14 lbs., tare 8 lbs. per cwt., tret and cloff as usual.
Ans. 69 cwt. 0 qr. 13 lbs.

39. RULE OF THREE.

(Where a knowledge of fractions is required.)

1. If the carriage of 2$\frac{3}{4}$ cwt. for 26 miles be 6s. 5d., what would be the charge for 1 cwt. for the same distance?

Cost for 2$\frac{3}{4}$ cwt. = 6s. 5d.

Taking the weight 4 times to get rid of the fraction,

Cost for 11 cwt. = 6s. 5d. × 4 = 25s. 8d.

∴ " 1 cwt. = $\frac{1}{11}$ of 25s. 8d. = 2s. 4d.

Or thus. $2\frac{3}{4} = \frac{11}{4}$: then

Cost for $\frac{11}{4}$ cwt. = 6s. 5d.

∴ " $\frac{1}{4}$ cwt. = $\frac{1}{11}$ of 6s. 5d. = 7d.

∴ " 1 cwt. = 7d. × 4 = 2s. 4d.

Or we may at once write down,

$$\text{Cost for 1 cwt.} = \frac{6s.\ 5d.}{2\frac{3}{4}} = \frac{6s.\ 5d \times 4}{11} = \&c.$$

2. If the carriage of $10\frac{2}{3}$ cwt. for $8\frac{1}{4}$ miles be 7s., to what distance should $7\frac{1}{2}$ cwt. be carried for the same money?

Distance for $10\frac{2}{3}$ cwt. = $8\frac{1}{4}$ miles.

∴ " 1 " = $8\frac{1}{4}$ miles × $10\frac{2}{3}$.

∴ " $7\frac{1}{2}$ " $= \dfrac{8\frac{1}{4} \times 10\frac{2}{3}}{7\frac{1}{2}}$ miles.

$$= \frac{33 \times 32 \times 2}{15 \times 4 \times 3} = \frac{176}{15} = 11\tfrac{11}{15} \text{ miles.}$$

3. If $2\frac{5}{8}$ cwt. cost £$1\frac{2}{3}$, what cost $20\frac{2}{5}$ cwt.?

By fractions. $2\frac{5}{8} = \frac{21}{8}$; $20\frac{2}{5} = \frac{102}{5}$; $1\frac{2}{3} = \frac{5}{3}$; then

$$\text{Cost } 20\tfrac{2}{5} = £\,\frac{1\frac{2}{3} \times 20\frac{2}{5}}{2\frac{5}{8}} = £\,\frac{\frac{5}{3} \times \frac{102}{5}}{\frac{21}{8}} = £\,\frac{5 \times 102 \times 8}{21 \times 3 \times 5}$$

$$= £12\ 19s.\ 0\tfrac{4}{7}d.$$

By decimals. $2\frac{5}{8} = 2{\cdot}625$; $20\frac{2}{5} = 20{\cdot}4$; then

$$\text{Cost } 20{\cdot}4 = £\,\frac{5 \times 20{\cdot}4}{3 \times 2{\cdot}625} = £\,\frac{102}{7{\cdot}875} = £12\ 19s.\ 0\tfrac{4}{7}d.$$

4. If 3 cwt. 2 qrs. cost £2 12s. 6d., what cost 2 cwt. 3 qrs.?

3 cwt. 2 qrs. = $3\frac{1}{2}$ cwt.; 2 cwt. 3 qrs. = $2\frac{3}{4}$ cwt.; then

$$\text{Cost } 2\tfrac{3}{4} \text{ cwt.} = £2\ 12s.\ 6d. \times \frac{2\frac{3}{4}}{3\frac{1}{2}} = £2\ 12s.\ 6d. \times \tfrac{11}{14} = £2\ 1s.\ 3d.$$

(Questions of this kind may be solved without fractions, as in Ex. 15, Art. 6)

5. If 2 cwt. 2 qrs. 14 lbs. cost 26s. 3d., how much may be got for 42s.?

2 cwt. 2 qrs. 14 lbs. = $2\frac{5}{8}$ cwt.; 26s. 3d. = $26\frac{1}{4}$s.; then

No cwt. for $26\frac{1}{4}$s. = $2\frac{5}{8}$.

∴ " 42s. $= \dfrac{2\frac{5}{8} \times 42}{26\frac{1}{4}} = \dfrac{21 \times 42}{210} = 4\frac{1}{5}$.

Or thus by decimals. $2\frac{5}{8} = 2{\cdot}625$; $26\frac{1}{4} = 26{\cdot}25$; then

$$\text{No. cwt. for 42s.} = \frac{2{\cdot}625 \times 42}{26{\cdot}25} = 4{\cdot}2 \text{ or } 4\tfrac{1}{5}.$$

6. If $5\frac{1}{2}$ cwt. cost £$8\frac{1}{4}$, what cost $3\frac{1}{5}$ cwt.? *Ans.* £4 16s.

7. If an ounce of silver cost $6\frac{1}{3}$s., what cost $8\frac{7}{16}$ oz.? *Ans.* £2 13s. $5\frac{1}{4}$d.

8. How many reams of paper can be bought for £60, at the rate of £30 12s. for 26 reams? *Ans.* $50\frac{50}{51}$.

9. If $\frac{5}{14}$ of a lb. cost $\frac{2}{9}$s., what cost $\frac{3}{10}$ of a cwt.? *Ans.* £1 0s. $10\frac{3}{4}$d.

10. If 6s. 3d. purchase $1\frac{2}{5}$ lbs. of tea, how much can be got for £7 15s.? *Ans.* 34 lbs. $11\frac{13}{15}$ oz.

11. How many yards of cloth can I buy for £114 6s. $3\frac{1}{2}$d. at the rate of 17s. 6d. for $2\frac{1}{8}$ yards? *Ans.* $277\frac{2087}{3360}$.

12. Bought $7\frac{1}{2}$ doz. of port at £1 12s. 6d. per doz., and 27 doz. 2 bottles of porter at 10s. 6d. per doz.; required the amount of the bill.
Ans. £26 9s.

13. What cost 622 lambs, at £9 8s. $1\frac{1}{2}$d. per score?
Ans. £292 10s. $8\frac{1}{4}$d.

14. If the rent of 6 acres 1 r. 15 p. be £14 4s. $4\frac{1}{2}$d., what will be the rent of 29 acres 3 r. 30 p. at the same rate? *Ans.* £67 2s. $0\frac{1}{4}$d.

15. If the rent of a farm of 99 acres 0 r. $32\frac{1}{2}$ p. be £370 7s. 2d., what is that per acre? *Ans.* £3 14s. 8d.

16. If the repairs of a street $\frac{3}{8}$ of a mile long be £3 14s. 9d., what part of the expense should be paid by the owner of a house having a frontage of $9\frac{1}{3}$ yds.? *Ans.* 1s. $0\frac{1}{2}$d.

17. Required the cost of 19 cwt. 3 qrs. 27 lbs., at £2 8s. 11d. for 1 cwt. $23\frac{1}{2}$ lbs. *Ans.* £40 8s. $3\frac{1}{2}$d.

18. A servant enters service on August 15th and quits it on Jan. 1st following, what ought he to receive at the yearly wages of 5 guineas?
Ans. £1 19s. $11\frac{3}{4}$d.

19. If 24·7 lbs. cost 14·8s. how many lbs. can be got for £12·6835?
Ans. 423·35465.

20. How much wheat will be required to sow a field containing 6·24 acres, allowing $2\frac{1}{2}$ pecks of wheat for every 3 roods? *Ans.* 20·8 pecks.

21. If the carriage of 17 cwt. for $16\frac{2}{3}$ miles cost £2, how far can I have $4\frac{1}{6}$ cwt. carried for the same money? *Ans.* 68 miles.

22. If a mass of silver be worth £7200 when silver is at 6s. 8d. per. oz., what is its worth when silver is at 6s. per oz.? *Ans.* £6480

23. If the population of a town be 5600, and the number of children at school at 630; what per cent. of the entire population were at school?
Ans. $11\frac{1}{4}$.

24. How many lbs. of tea can be bought for £14 16s. $10\frac{1}{2}$d., when $7\frac{3}{8}$ cwt. cost £436 8s. $1\frac{1}{2}$d.? *Ans.* $28\frac{2}{21}$ lbs.

25. What is the value of 3 cwt. 3 qrs. 23 lbs. of sugar, at 2s. $2\frac{1}{4}$d. for $5\frac{1}{2}$ lbs.? *Ans.* £8 16s. $2\frac{1}{4}$d.

26. Bought $\frac{5}{8}$ of a property, and sold $\frac{3}{5}$ of my share for £420; what was the whole property worth?

Portion sold $= \frac{3}{5}$ of $\frac{5}{8} = \frac{3}{8}$; then we have—

Value of $\frac{3}{8} =$ £420.

$\therefore$,, $\frac{1}{8} = £\frac{420}{3} =$ £140.

$\therefore$,, 1 or the whole $=$ £140 $\times 8 =$ £1120.

27. What length of paper $1\frac{3}{4}$ feet wide will cover a wall 15 feet by $9\frac{1}{3}$ feet?

Bringing the fractions to the same denominator, we find

$1\frac{3}{4} = \frac{7}{4} = \frac{21}{12}$; $9\frac{1}{3} = \frac{28}{3} = \frac{112}{12}$; then

Length of paper $\frac{112}{12}$ ft. wide $=$ 15 ft.

$\therefore$,, $\frac{1}{12}$,, $=$ 15 ft. $\times$ 112.

$\therefore$,, $\frac{21}{12}$,, $= \frac{15 \times 112}{21}$ ft. $=$ 80 ft.

Or we may at once write down,

$$\text{No. ft. } 1\tfrac{3}{4} \text{ ft. wide} = \frac{15 \times 9\frac{1}{3}}{1\frac{3}{4}} = \frac{15 \times \frac{28}{3}}{\frac{7}{4}} = \frac{15 \times 28 \times 4}{7 \times 3} = 80.$$

28. If the sixpenny loaf weigh 3 lbs. when wheat is 6s. 6d. per bushel, what should it weigh when wheat is 6s. per bushel?

Weight of loaf when wheat is $6\frac{1}{2}$s. = 3 lbs.

∴ „ „ 1s. = 3 lbs. × $6\frac{1}{2}$.

∴ „ „ 6s. = $\dfrac{3 \times 6\frac{1}{2}}{6}$ lbs. = $3\frac{1}{4}$ lbs.

29. A bankrupt's effects amounted to £490 5s., with which he paid 13s. 4d. in the pound; required the amount of his debts.

Debt when the payment is £$\frac{2}{3}$ = £1.

∴ „ £1 = £$\frac{3}{2}$.

∴ „ £$490\frac{1}{4}$ = £$\frac{3}{2}$ × $490\frac{1}{4}$ = £735 7s. 6d.

30. If I expend £523 12s. in shares at $95\frac{1}{5}$ per cent., and sell at $86\frac{2}{3}$, how much do I lose?

Loss on £$95\frac{1}{5}$ = £$95\frac{1}{5}$ — £$86\frac{2}{3}$ = £$8\frac{8}{15}$.

∴ „ £$523\frac{3}{5}$ = £$8\frac{8}{15}$ × $\dfrac{523\frac{3}{5}}{95\frac{1}{5}}$ = £46 18s. 8d.

31. Supposing a man's wages to be regulated by the price of corn, and that he receives 2s. 6d. a day when the corn is 5s. 10d. per bushel, what will he receive when the corn is 4s. 7d. per bushel?

Wages when corn is 70d. per bushel = 2s. 6d.

∴ „ 55d. „ = 2s 6d. × $\frac{55}{70}$ = 1s. $11\frac{4}{7}$d.

32. If the shadow of a tower measure 64·4 feet, and that of an upright post 6 feet high, 4·2 feet, required the height of the tower.

Height when the shadow is 4·2 feet = 6 ft.

∴ „ „ 1 „ = $\dfrac{6}{4{\cdot}2}$ ft.

∴ „ „ 64·4 „ = $\dfrac{6 \times 64{\cdot}4}{4{\cdot}2}$ ft. = 92 ft.

33. I lent £36 for 5 months, how long should £25 be lent to me in return?

No. months for a loan of £36 = 5.

∴ „ „ £1 = 5 × 36.

∴ „ „ £25 = $\dfrac{5 \times 36}{25}$ = $7\frac{1}{5}$.

34. How many lbs. at 1s. 8d. are equal in value to 70 lbs. at 2s. 4d. per lb.?

No. lbs. at 28d. = 70.

∴ „ 1d. = 70 × 28.

∴ „ 20d. = $\dfrac{70 \times 28}{20}$ = 98.

Or thus. Value 70 lbs. = 28d. × 70 = 1960d.

∴ No. lbs. required = $\frac{1960}{20}$ = 98.

35. A ship has 18 months' provisions for 15 men; how long would the provisions last 24 men?

No. months' provisions for 15 men = 18.

∴ „ 1 „ = 18 × 15.

∴ „ 24 „ = $\dfrac{18 \times 15}{24}$ = $11\frac{1}{4}$.

36. How many men would the provisions of the last example serve for 12 months?

No. men supplied for 18 months $= 15$.

$\therefore$,, 1 ,, $= 15 \times 18$.

$\therefore$,, 12 ,, $= \dfrac{15 \times 18}{12} = 22\frac{1}{2}$.

37. If a train run $5\frac{1}{2}$ miles in 14 min. 6 sec.; in what time would it run $10\frac{1}{4}$ miles at the same rate?

No. min. to run $5\frac{1}{2}$ miles $= 14\frac{1}{10}$.

$\therefore$,, $10\frac{1}{4}$,, $= \dfrac{14\frac{1}{10} \times 10\frac{1}{4}}{5\frac{1}{2}} = 26$ min. $16\frac{7}{11}$ sec.

Or thus. No miles in $14\frac{1}{10}$ min. $= 5\frac{1}{2}$.

$\therefore$,, 141 ,, $= 5\frac{1}{2} \times 10 = 55$.

$\therefore$,, 1 ,, $= \frac{55}{141}$.

$\therefore$ No. min. to run $10\frac{1}{4}$ miles $= 10\frac{1}{4} \div \frac{55}{141} = \frac{41}{4} \times \frac{141}{55} =$ as before.

38. Paid 30s. for poor's rate at 1s. 6d. in the pound; what is the rent of my house, supposing the rate to be charged on $\frac{2}{3}$ of the actual rent?

No. pounds rateable rent $=$ 30s. $\div$ $1\frac{1}{2}$s. $= 20$.

But this £20 is two-thirds of my actual rent.

$\therefore$ $\frac{2}{3}$ rent $=$ £20; $\therefore$ $\frac{1}{3}$ rent $=$ £10; $\therefore$ rent $=$ £30.

39. If a rate of 2s. 3d. in the pound be made on $\frac{2}{3}$ of the rental of a parish, required the rate per pound on the actual rental, in order to produce the same amount.

Here the rate per pound must be less when the whole rental is taken.

Rate taking $\frac{2}{3}$ of the rental $=$ 2s. 3d.

$\therefore$,, $\frac{1}{3}$,, $=$ 2s. 3d. $\times$ 2.

$\therefore$,, the whole rental $=$ 2s. 3d. $\times \frac{2}{3} =$ 1s. 6d.

40. Paid £6 18s. for taxes; what is the rent of my property, allowing 10d. income tax on each pound of the actual rent, and 4s. 6d. in the pound on $\frac{2}{3}$ of the actual rent for all other taxes?

Tax on $\frac{2}{3}$ rental $=$ 4s. 6d.; $\therefore$ Tax on whole rental $=$ 3s.;

$\therefore$ Total tax per pound $=$ 3s. $+$ 10d. $=$ 3s. 10d.

$\therefore$ No. pounds rent $=$ £6 18s. $\div$ 3s. 10d. $=$ £36.

41. Two couriers pass through a place at an interval of 2 hours, travelling at the rates of $8\frac{1}{2}$ and 10 miles per hour respectively; in what time will the second courier overtake the first?

Distance of the first in advance of the second $= 8\frac{1}{2} \times 2 = 17$ m.

Rate per hour at which the 2nd approaches the 1st. $= 10 - 8\frac{1}{2} = 1\frac{1}{2}$ m.

$\therefore$ No. hours required $= 17 \div 1\frac{1}{2} = 11\frac{1}{3} =$ 11 h. 20 min.

42. A wheel revolves 22 times in $3\frac{1}{2}$ minutes, how often will it revolve in an hour?

No. revolutions per min. $= 22 \div 3\frac{1}{2} = \frac{44}{7}$.

$\therefore$,, per hour $= \frac{44}{7} \times 60 = 377\frac{1}{7}$.

43. The flash of a cannon was seen 5 seconds before the report was heard; required the distance of the cannon, the velocity of sound being 1142 feet per second.

No. feet sound travels in 1 sec. $= 1142$.

$\therefore$,, ,, 5 ,, $= 1142 \times 5 = 5710$.

44. Bought $\frac{3}{4}$ of a property, and sold $\frac{1}{4}$ of my share for £120; required the value of the whole property. *Ans.* £640.

45. If it requires $2\frac{7}{8}$ yards of cloth, $\frac{3}{4}$ of a yard wide, to make a coat, how many yards will it take when the cloth is $1\frac{1}{8}$ yards wide? *Ans.* $1\frac{11}{12}$.

46. If 39 yards of carpeting, 32 inches wide, cover a floor, how many yards will be required of carpet which is only 26 inches wide? *Ans.* 48.

47. If a train runs $16\frac{1}{2}$ miles in 36 min. 15 sec., in what time will it run 24 miles $5\frac{1}{2}$ furlongs? *Ans.* 54 min. $14\frac{23}{88}$ sec.

48. A bankrupt pays his creditors 6s. 8d. in the pound, paying them £245; required his debt. *Ans.* £735.

49. A bankrupt owes his creditors £1920, and pays only £252; how much does he pay them per pound? *Ans.* 2s. $7\frac{1}{2}$d.

50. If 48 tenpence-loaves can be got from a quarter of flour, how many shilling-loaves should be got from it? *Ans.* 40.

51. If the 4 lb. loaf cost 9d. when the flour is 4s. per stone, what should the same weight of loaf cost when the flour is 3s. 6d.? *Ans.* $7\frac{7}{8}$d.

52. How many lbs. at 1s. 8d. are equal in value to 70 lbs. at 2s. 4d. per lb.? *Ans.* 98.

53. How many yards of carpet, $\frac{3}{4}$ of a yard wide, will cover a floor 27 feet by 20; and what would be the cost of the carpet at 3s. 6d. per yd.? *Ans.* 80; £14.

54. The flash of the lightning was seen 9 seconds before the thunder was heard; required the distance of the thunder. (See Ex. 43.) *Ans.* 1·946 miles.

55. A man can plough 1 acre 3 roods in 10 hours, what time would he take to plough 6 acres 20 perches? *Ans.* 35 hours.

56. If I pay £3 13s. 6d. for income tax yearly at 7d. in the pound, what is my income? *Ans.* £126.

57. Required the income tax on £250 10s. at 7d. per £1. *Ans.* £7 6s. $1\frac{1}{2}$d.

58. The expenses of a parish are £104 5s., and the rateable rent £1668; how much in the pound must be levied to pay the expenses? *Ans.* 1s. 3d.

59. How many casks of raisins, each 3 cwt., can I buy for £49, at £8 3s. 4d. for 4 cwt.? *Ans.* 8 casks.

60. Bought 60 yards at 2s. for 5 yards, and 72 yards at 2s. 3d. for 9 yards; required the cost of the whole. *Ans.* £2 2s.

61. Bought 13·5 cwt. of sugar for £32·25, how much can be got for £21·5? *Ans.* 9 cwt.

62. If $9\frac{1}{3}$ oz. of tea cost $17\frac{15}{16}$s., what cost $17\frac{1}{12}$ lbs.? *Ans.* £26 18s. $1\frac{1}{2}$d.

63. If 8·9375 cwt. cost £4 9s. $4\frac{1}{2}$d., how much may be bought for £11 5s.? *Ans.* 22 cwt. 2 qrs.

64. How much should a person spend per month, if he wishes to save $\frac{1}{4}$ of a yearly income of £240?

Sum to spend in 12 months = £240 — $\frac{1}{4}$ of £240 = £180.

∴ " 1 " = $\frac{1}{12}$ of £180 = £15.

65. How much should a person spend in 146 days, if he wishes to save every year £12 out of an income of £162? *Ans.* £60.

66. Required the same as in Ex. 38, when the rate is 6d. in the pound. *Ans.* £90.

67. Required the same as in Ex. 39, when the rate is 15d. in the pound. *Ans.* 10d.

68. Required the same as in Ex. 40, when the income tax is 14d. in the pound and the other taxes 3s. 10d. *Ans.* £19 19s. $0\frac{26}{83}$d.

69. If a man can finish $\frac{2}{7}$ of a certain piece of work in 9 hours, in what time would he complete the remainder?

Remaining part $= 1 - \frac{2}{7} = \frac{5}{7}$; then we have

No. hours to finish $\frac{2}{7} = 9$.

$\therefore$ „ $\frac{5}{7} = \frac{9}{2} \times 5 = 22\frac{1}{2}$.

70. If a man can finish $\frac{1}{8}$ of a certain piece of work in 5 hours, in what time would he finish $\frac{1}{3}$ of the remainder? *Ans.* $11\frac{2}{3}$ hours.

71. A man can complete a certain job in 10 days; after being at work for 2 days, he was joined by two other men; in what time would they finish the remainder? *Ans.* $2\frac{2}{3}$ days.

72. A man can build $3\frac{1}{3}$ yds. of wall in $5\frac{1}{2}$ days, in what time would he build $23\frac{1}{3}$ yds.? *Ans.* $38\frac{1}{2}$ days.

73. Required the charges on a property of £24 rent, allowing 14d. income tax on every pound of the actual rent, and 5s. 6d. in the pound on $\frac{2}{3}$ of the rent for all other taxes. *Ans.* £5 16s.

74. How many dollars, each 4s. 6d., are equal in value to 42 coins 7s. 6d. each?

No. coins $7\frac{1}{2}$s. each $= 42$.

$\therefore$ „ $\frac{1}{2}$ „ $= 42 \times 15$.

$\therefore$ „ $4\frac{1}{2}$ „ $= \dfrac{42 \times 15}{9} = 70$.

Or thus. Value of the money $= 7\frac{1}{2}$s. $\times\ 42 = 315$s.

$\therefore$ No. dollars $= 315 \div 4\frac{1}{2} = 70$.

75. How many half-crowns are equal in value to 50 two-shilling pieces? *Ans.* 40.

76. By using a false measure, a person sells $7\frac{3}{8}$ pints to the gallon; of how much money does he defraud his customers in selling 160 gallons of wine at 18s. 6d. per gallon?

Fraud in 1 gallon = 1 gallon — $7\frac{3}{8}$ pints $= \frac{5}{8}$ pints.

$\therefore$ „ 160 „ $= \frac{5}{8} \times 160 = 100$ pints.

Value 100 pints = 18s. 6d. $\times\ \frac{100}{8} =$ £11 11s. 3d.

77. Of how much in money does a person defraud his customers in selling 1 cwt. of sugar at 4d. per lb., if he uses a false weight of $15\frac{11}{16}$ oz. for 1 lb.? *Ans.* $8\frac{3}{4}$d.

78. A person gave £82 for 74 gallons of spirits, to which he added 6 gallons of water; at what price per gallon must he sell the mixture to gain £8?

No. gallons mixture $= 74 + 6 = 80$; then

Selling price of 80 gallons = £82 + £8 = £90.

$\therefore$ „ 1 „ = £90 $\div$ 80 = £1 2s. 6d.

79. What would be the selling price per gallon, in the last example, if 16 gallons of water be added, with a gain of £12 10s.? *Ans.* 21s.

80. The circumference of a coach wheel is 16 feet, how many revolutions will it make in a mile? *Ans.* 330.

81. Supposing the coach, of the last example, to travel at the rate of 10 miles per hour, how many revolutions will the wheel make per minute? *Ans.* 55.

82. A man, travelling 8 hours a day, completes a journey in $6\frac{1}{2}$ days; how many hours a day must he walk to perform it in 5 days?

No. hours to complete it in $6\frac{1}{2}$ days $= 8$.

$\therefore$ „ „ 1 „ $= 8 \times 6\frac{1}{2} = 52$.

$\therefore$ „ „ 5 „ $= \frac{1}{5}$ of $52 = 10\frac{2}{5}$

Or thus. No hours on the journey $= 8 \times 6\frac{1}{2} = 52$;

But he has to complete the journey in 5 days;

$\therefore$ No. hours per day $= 52 \div 5 = 10\frac{2}{5}$.

83. Supposing the man, of the last example, to travel at the rate of $3\frac{1}{4}$ miles per hour; what would be the length of the journey? At what rate per hour must he travel to complete this journey in $5\frac{1}{2}$ days of 10 hours long? *Ans.* 169 miles; $3\frac{4}{33}$ miles.

84. A vessel sails at the rate of 75 miles in 10 hours; in what time will she make a voyage of 1500 miles? *Ans.* 8 days 8 hours.

85. A watch gains 7 minutes in 24 hours, while another watch loses $5\frac{1}{2}$ minutes in the same time; if both be set right at the same time, what will be the difference of time between them after the lapse of 60 hours?

Difference in 24 hours $= 7 + 5\frac{1}{2} = 12\frac{1}{2}$ minutes.

$\therefore$ „ 60 „ $= 12\frac{1}{2} \times \frac{60}{24} = 31\frac{1}{4}$ minutes.

86. Required the same as in the last example, when the first gains 35 seconds in 4 hours, and the other gains 15 seconds in the same time. *Ans.* 5 minutes.

87. A fast train starts 2 hours after a slow one. In what time will the former overtake the latter, allowing their speeds to be 30 and 20 miles per hour respectively? What distance will each have gone?

The distance which the fast train has to make up $= 20 \times 2 = 40$ miles.

Distance gained by the fast train per hour $= 30 - 20 = 10$ miles.

$\therefore$ No. hours required $= \frac{40}{10} = 4$.

And distance gone $= 30$ miles $\times 4 = 120$ miles.

88. Required the same as in the last example, when the speeds are 32 and 24 miles respectively. *Ans.* 6 hours; 192 miles.

89. In what time would the fast train, of Ex. 87, be 55 miles in advance of the other? *Ans.* $9\frac{1}{2}$ hours.

90. At what time will the two pointers of a watch be together between IV. and V. o'clock?

When the short pointer is at IV. the long pointer is at XII.—that is, the former is 20 minute spaces in advance of the latter.

As the long pointer moves 12 times as fast as the short one, so, therefore, while the former moves over 12 minute spaces, the latter only moves over 1; and, therefore, in 12 minutes of time the long pointer will gain 11 minute spaces on the short pointer: hence we write—

Time to gain 11 minute spaces $= 12$ min.

$\therefore$ „ 20 „ $= 12$ min. $\times \frac{20}{11} = 21\frac{9}{11}$ min.

That is, the time will be $21\frac{9}{11}$ min. past 4.

91. At what time will the two pointers of a watch be together between VI. and VII. o'clock? *Ans.* $32\frac{8}{11}$ min. past 6.

40. COMPOUND RULE OF THREE.

1. If 5 firkins of butter, at 8d. a lb., cost £9, what will be the cost of 7 firkins at 10d. a lb.?

Here, as *cost* is required, we write down, from the data of the question, the following equality for cost:

Cost 5 firkins at 8d. a lb. = £9.

$\therefore$ „ 1 „ 8d. „ $= £\frac{9}{5}$,

$\therefore$ „ 1 „ 1d. „ $= £\frac{9}{5 \times 8}$

$\therefore$ „ 7 „ 1d. „ $= £\frac{9 \times 7}{5 \times 8}$

$\therefore$ „ 7 „ 10d. „ $= £\frac{9 \times 7 \times 10}{5 \times 8} = £15\ 15s.$

Omitting the second and fourth steps, the student may readily write down the third equality, giving the value of a unit of quantity at a unit of price, and from it the fifth equality, giving the value of the quantities required in the question.

2. If the wages of 26 men for 6 days be £14, what will be the wages of 36 for $10\frac{1}{2}$ days?

Here, as *wages* are required, we write down the following equality for *wages:*

Wages 26 men for 6 days = £14.

$\therefore$ „ 1 „ 1 „ $= £\frac{14}{26 \times 6}$

$\therefore$ „ 36 „ $10\frac{1}{2}$ „ $= £\frac{14 \times 36 \times 10\frac{1}{2}}{26 \times 6} = £33\ 18s.\ 5\frac{7}{13}d.$

Or thus. No. days' work in the 1st case $= 26 \times 6 = 156$.

„ 2nd „ $= 36 \times 10\frac{1}{2} = 378$.

Wages for 156 days = £14.

$\therefore$ Wages for 378 days $= £\frac{14 \times 378}{156} = £\frac{7 \times 63}{13} =$ as before.

3. If 6 yards of cloth, 3 quarters wide, cost £$9\frac{1}{2}$, what will be the cost of 4 yards, 5 quarters wide? *Ans.* £10 11s. $1\frac{1}{3}$d.

4. If 5 horses can be kept 7 days for £3 5s., what will be the cost of keeping 9 horses for 21 days? *Ans.* £17 11s.

5. If 6 horses can be kept 20 days for £15, how many horses can be kept 16 days for £18?

Here, as the *number of horses* is required, we write down the following equality for the *number of horses:*

No. horses kept 20 days for £15 = 6.

$\therefore$ „ 1 „ $£1 = \frac{6 \times 20}{15}$

$\therefore$ „ 16 „ $£18 = \frac{6 \times 20 \times 18}{15 \times 16} = 9.$

The second equality is obtained from the first in the following manner: The number of horses kept 1 day for £15 will be 20 times the number kept 20 days; and then the number kept 1 day for £1 will be the 15th part of the number kept 1 day for £15: hence we multiply 6 by 20, and divide by 15.

In the third equality we obtain the number of horses required in the question: The number of horses kept for £18 will be 18 times the number kept for £1; and then the number kept 16 days for this money will be the 16th part of the number kept only 1 day; hence we multiply the result last found by 18, and divide by 16.

Or thus. Cost 1 horse per day $= \dfrac{15}{6 \times 20}$

$\therefore$ Cost 1 horse for 16 days $= \dfrac{15 \times 16}{6 \times 20}$

$\therefore$ No. horses required $= £18 \div £\,\dfrac{15 \times 16}{6 \times 20} = \dfrac{18 \times 6 \times 20}{15 \times 16} = 9.$

6. If 6 men working for 9 days earn £9 6s., how many men working for 2 days will earn £3 2s.? *Ans.* 9 men.

7. If 3 horses plough a field of 2 acres in $1\frac{1}{2}$ days, in how many days will 21 horses plough a field of 49 acres?

No. days for 3 horses to plough 2 acres $= 1\frac{1}{2}$.

$\therefore$ „ 1 „ 1 „ $= \dfrac{1\frac{1}{2} \times 3}{2}$

$\therefore$ „ 21 „ 49 „ $= \dfrac{1\frac{1}{2} \times \cancel{3} \times \overset{7}{\cancel{49}}}{2 \times \underset{7}{\cancel{21}}} = 5\frac{1}{4}.$

8. If 3 men can build 15 yds. of brickwork in 7 days, how many days would it take 5 men to build 40 yds.? *Ans.* $11\frac{1}{5}$.

9. If 8 horses can be kept 15 days for £20, how many days can 18 horses be kept for £27? *Ans.* 9 days.

10. If 640 tiles 12 inches long and 9 inches broad pave a floor, how many tiles 10 inches long and 8 inches broad would be required?

No. tiles 12 in. by 9 in. $= 640$.

„ 1 „ 1 „ $= 640 \times 12 \times 9$.

„ 10 „ 8 „ $= \dfrac{640 \times 12 \times 9}{10 \times 8} = 864.$

Or thus. No. sq. inches in the floor $= 12 \times 9 \times 640$.

$\therefore$ No. sq. inches in 1 tile $= 10 \times 8$.

$\therefore$ No. tiles required $= \dfrac{12 \times 9 \times 640}{10 \times 8} =$ as before.

11. If 35 yds. of carpet cover a floor 20 feet long and 14 feet broad, how many yards of carpet of the same width will cover a floor 18 feet long and 16 feet broad? *Ans.* 36.

12. If a plate of copper, 5 inches long and 4 inches broad, weigh $15\frac{3}{4}$ oz., what will be the weight of a plate of the same thickness, 2 ft. 6 in. long and 8 in. broad? *Ans.* 189 oz.

13. If a plot of ground, 136 feet long and 30 feet broad, cost £51, what will be the cost of a plot 210 feet long and 44 feet broad?

Cost 136 feet by 30 feet = £51.

$$\therefore \quad ,, \ 1 \quad ,, \ 1 \ ,, = £\frac{51}{136 \times 30}$$

$$\therefore \quad ,, \ 210 \quad ,, \ 44 \ ,, = £\frac{51 \times 210 \times 44}{136 \times 30} = £115 \ 10s.$$

Or thus. No. sq. ft. in the 1st plot = 136 × 30.

,, 2nd ,, = 210 × 44.

$$\therefore \text{ Cost of 1 sq. ft.} = £\frac{51}{136 \times 30}$$

$$\therefore \quad ,, \ 210 \times 44 \ ,, = £\frac{51 \times 210 \times 4}{136 \times 30} \text{ as before.}$$

14. If a pavement, 35 yds. long and 3 yds. wide, cost £25, what will be the cost of a pavement 33 yds. by $3\frac{1}{2}$ yds.? *Ans.* £27 10s.

15. If 140 yds. of cloth, $\frac{7}{8}$ wide, clothe 50 men, how many yds., $\frac{5}{4}$ wide, will be required to clothe 60 men?

No. yds. $\frac{7}{8}$ wide for 50 men = 140.

$$,, \ \tfrac{1}{8} \quad ,, \ 1 \ ,, = \frac{140 \times 7}{50}$$

$$,, \ \tfrac{10}{8} \quad ,, \ 60 \ ,, = \frac{140 \times 7 \times 60}{50 \times 10} = 117\tfrac{3}{5}.$$

16. If 16 yds. of cloth, $\frac{3}{4}$ wide, cover 24 chairs, how many yards $\frac{2}{3}$ wide will be required to cover 30 chairs? *Ans.* $22\frac{1}{2}$.

17. 20 men are set to build a wall 360 yds. long, which they were expected to finish in 30 days; but at the end of 10 days it was found that they had only completed 90 yards; how many more men must be employed to finish it in the proposed time?

Time remaining = 30 — 10 = 20 days; No. yds. remaining = 360 — 90 = 270.

No. men to build 90 yds. in 10 days = 20.

$$\therefore \quad ,, \ 1 \ ,, \ 1 \ ,, = \frac{20 \times 10}{90}$$

$$\therefore \quad ,, \ 270 \ ,, \ 20 \ ,, = \frac{20 \times 10 \times 270}{90 \times 20} = 30.$$

Therefore, the additional number of men = 30 — 20 = 10.

18. In what time would 30 men (see last Ex.) build the whole wall?

No. days for 20 men to build 90 yds. = 10.

,, 10 ,, 360 yds. = 10 × 4 × 2 = 80.

,, 30 ,, 360 yds. = $\frac{1}{3}$ of 80 = $26\frac{2}{3}$.

Or thus. No. days' work in building 90 yds. = 20 × 10.

∴ ,, ,, 360 yds. = 20 × 10 × 4 = 800.

∴ No. days required = $\frac{800}{30}$ = as before.

19. If 6 men can build a wall in $18\frac{1}{2}$ days of $10\frac{1}{4}$ hours each, how many days would it take 5 men to build $3\frac{3}{4}$ as much, working for 9 hours per day? *Ans.* $94\frac{13}{16}$ days.

20. If the carriage of 8 cwt. for 10 miles cost 6s., what will 17 cwt. for 15 miles cost?

Cost 8 cwt. for 10 miles = 6s.

$$\therefore \quad \text{Cost 17 cwt. for 15 miles} = \frac{6 \times 17 \times 15}{8 \times 10} = 19s.\ 1\tfrac{1}{2}d.$$

21. Two women are employed in making shirts. Now the first woman can make 3 shirts whilst the second is making 2; and working together they can make 75 shirts in 30 days. What time will each take in making a shirt; and supposing the first woman to work for 10 days and the second for 20 days, how many shirts will they make in the time?

Here, since $3 + 2 = 5$, we shall first find the time which they take in making 5 shirts.

No. days in making 75 shirts = 30.

$\therefore$,, 5 ,, = 2.

That is, in 2 days the first woman will make 3 shirts, and the second woman 2 shirts; therefore, the former will make a shirt in $\frac{2}{3}$ of a day, and the latter in 1 day.

No. made by the 1st in 2 days = 3.

$\therefore$,, ,, 10 ,, $= 3 \times 5 = 15$.

And ,, 2nd in 20 ,, $= 2 \times 10 = 20$.

And the number made by both in the specified times $= 15 + 20 = 35$.

22. Required the same as in the last example, supposing the first woman to make 4 shirts whilst the second makes 3.

Ans. $\frac{7}{10}$ and $\frac{14}{15}$ of a day; $35\frac{4}{7}$.

23. If a man can perform the work of 2 women or of 3 boys; and if 4 men, 10 women, and 9 boys can finish a certain work in 60 days, in what time would 3 men, 8 women, and 6 boys finish the same?

Here, converting the women and boys into men, we find

4 men + 10 women + 9 boys $= (4 + \frac{10}{2} + \frac{9}{3})$ men = 12 men; and
3 men + 8 women + 6 boys $= (3 + \frac{8}{2} + \frac{6}{3})$ men = 9 men; then

No. days for 12 men = 60.

$$\therefore \quad ,, \quad 9 \quad ,, \quad = \frac{60 \times 12}{9} = 80. \quad \textit{Ans.}$$

24. In what time would 2 men, 5 women, and 12 boys finish the work in the last example? *Ans.* $84\frac{12}{17}$ days.

25. Required the same as in Ex. 23, supposing that 3 men can do the work of 5 women, and 2 women can do the work of 3 boys.

Ans. 80 days.

26. If £246 in trade gain £24 in 7 months, what will be the gain of £369 in $1\frac{3}{4}$ months?

Gain on £246 in 7 months = £24.

$$,, \quad £1 \text{ in } 1 \quad ,, \quad = £\frac{24}{246 \times 7}$$

$$,, \quad £369 \text{ in } 1\tfrac{3}{4} \quad ,, \quad = £\frac{24 \times 369 \times 1\frac{3}{4}}{246 \times 7} = £9.$$

27. If £266 in trade gain £$7\frac{3}{5}$ in 3 months, what sum will gain £15 in 9 months? *Ans.* £175.

28. If 33 men dig a trench 300 yards long, 10 yards wide, and 2 yards deep, in 6 days of 9 hours each; how many men will be required

to dig a trench 400 yards long, 5 yards wide, and $1\frac{1}{2}$ yards deep, in 3 days of $8\frac{1}{4}$ hours each?

No. men to dig

300 yds. long, 10 wide, 2 deep, in 6 d. of 9 h. = 33.

1 „ 1 „ 1 „ in 1 d. of 1 h. $= \frac{33 \times 6 \times 9}{300 \times 10 \times 2}$

400 „ 5 „ $1\frac{1}{2}$ „ in 3 d. of $8\frac{1}{4}$ h. $= \frac{33 \times 6 \times 9 \times 400 \times 5 \times 1\frac{1}{2}}{300 \times 10 \times 2 \times 3 \times 8\frac{1}{4}}$

$= 36.$

Or thus. No. cubic yards in the first trench $= 300 \times 10 \times 2 = 6000.$

„ second „ $= 400 \times 5 \times 1\frac{1}{2} = 3000.$

No. men to dig 6000 c. yds. in 54 hours = 33.

∴ „ 3000 „ $24\frac{3}{4}$ „ $= \frac{33 \times 54}{2 \times 24\frac{3}{4}} = 36.$

29. If 7 men set up a work of 5 sheets in 9 days of 10 hours each, how many men will be required to set up a work of 20 sheets in 8 days of 9 hours each? *Ans.* 35.

30. If a block of stone, 6 ft. long, 2 ft. broad, and $1\frac{1}{2}$ ft. thick, weigh 21 cwt., what will be the weight of a block of the same kind of stone, 4 ft. long, 3 ft. broad, and $1\frac{1}{4}$ ft. thick?

Wt. of block 6 ft. long, 2 ft. broad, $1\frac{1}{2}$ ft. thick = 21 cwt.

∴ „ 1 „ 1 „ 1 „ $= \frac{21}{6 \times 2 \times 1\frac{1}{2}}$ cwt.

∴ „ 4 „ 3 „ $1\frac{1}{4}$ „ $= \frac{21 \times 4 \times 3 \times 1\frac{1}{4}}{6 \times 2 \times 1\frac{1}{2}}$

$= 17\frac{1}{2}$ cwt.

Or thus. No. cubic feet in the 1st block $= 6 \times 2 \times 1\frac{1}{2} = 18.$

„ 2nd „ $= 4 \times 3 \times 1\frac{1}{4} = 15;$

Weight of 18 cubic feet = 21 cwt.

∴ „ 15 „ $= \frac{21 \times 15}{18}$ cwt. $= 17\frac{1}{2}$ cwt.

31. Required the weight of a cubic foot of the stone of the last example. *Ans.* $130\frac{2}{3}$ lbs.

32. Required the weight of a plate of copper, 1 ft. 6 in. long, 9 in. broad, and 2 in. thick, supposing a cubic foot to weigh 8840 oz. *Ans.* $1657\frac{1}{2}$ oz.

33. If an engine can raise 640 gallons of water in 1 hour 20 minutes, from a pit 90 fathoms in depth, how many gallons would it raise in 24 hours from a depth of 50 fathoms?

No. gal. in $\frac{4}{3}$ hours from 90 fathoms = 640.

„ 1 „ 1 „ $= \frac{640 \times 90 \times 3}{4}$

„ 24 „ 50 „ $= \frac{640 \times 90 \times 3 \times 24}{4 \times 50} = 20736.$

34. How long would the engine of the last example be in raising 2700 gallons from a depth of 80 fathoms.

No. hours raising 640 gal. from 90 fathoms $= \frac{4}{3}$.

$$\text{,,} \quad 1 \quad \text{,,} \quad 1 \quad \text{,,} \quad = \frac{4}{3 \times 640 \times 90}$$

$$\text{,,} \quad 27000 \quad \text{,,} \quad 80 \quad \text{,,} \quad = \frac{4 \times 27000 \times 80}{3 \times 640 \times 90} = 50.$$

35. If a man can pump 6 gallons of water in 5 minutes, from a well 36 feet deep, how many gallons would he pump in 10 hours when the depth is 48 feet? *Ans.* 540.

36. How long would the man of the last example be in pumping 3600 gallons from a depth of 45 feet? *Ans.* $62\frac{1}{2}$ hours.

37. Two men start a job, and after working for 3 days, of 10 hours each, they were joined by another man, and then, working together, they finished the remainder in 5 days of 9 hours each; in how many days, of 12 hours each, could 2 men have finished the whole?

No. hours work of 1 man in the job $= 3 \times 10 \times 2 + 5 \times 9 \times 3 = 195$.

But the job is to be done by 2 men working 12 hours per day;

$\therefore$ No. hours work per day $= 12 \times 2 = 24$.

$\therefore$ No. days required $= \frac{195}{24} = 8\frac{1}{8}$.

38. How many hours per day (see last Ex.) would the 2 men have to work so as to finish the whole in 8 days? *Ans.* 12 h. $11\frac{1}{4}$ min.

39. If 15 loaves last a family of 6 persons for 7 days, how long will 35 loaves last 9 persons?

No. days 15 loaves last 6 persons $= 7$.

$$\text{,,} \quad 1 \quad \text{,,} \quad 1 \quad \text{,,} \quad = \frac{7 \times 6}{15}$$

$$\text{,,} \quad 35 \quad \text{,,} \quad 9 \quad \text{,,} \quad = \frac{7 \times 6 \times 35}{15 \times 9} = 10\frac{8}{9}.$$

40. If 20 loaves last 7 persons for 8 days, how many loaves will 9 persons consume in 6 days? *Ans.* $19\frac{2}{7}$.

41. The provisions of a garrison were sufficient to last 1200 men for 12 months; but at the end of 3 months the garrison was increased by 300 men, and 2 months after that by another reinforcement of 200 men. What time would the provisions last the whole?

No. months' provisions for 1 man $= 1200 \times 12 = 14400$.

No. months' provisions consumed at the end of 5 months:

$= 1200 \times 5 + 300 \times 2 = 6600$.

No. months' provisions remaining $= 14400 - 6600 = 7800$.

Now we have to find the time which the remaining provisions will last 1700 men or $(1200 + 300 + 200)$ men.

No. months $= \frac{7800}{1700} = 4\frac{10}{17}$.

$\therefore$ Total number of months $= 5 + 4\frac{10}{17} = 9\frac{10}{17}$. *Ans.*

42. Required the same as in the last example, supposing the provisions at first were sufficient to last 1800 men for 10 months.

Ans. $8\frac{15}{23}$ months.

43. The provisions of a garrison were sufficient to last 1500 men for 12 months, but at the end of 2 months the garrison was increased by 500 men. How long did the provisions last? *Ans.* $9\frac{1}{2}$ months.

44. A ship's crew have provisions for 100 days, allowing each man daily $2\frac{1}{2}$ lbs. of bread and $\frac{3}{4}$ lb. of beef; after 40 days it was found that they must yet remain 80 days at sea; to what rations per day must the

crew be reduced for the remainder of the voyage? *Ans.* $1\frac{7}{8}$ lbs. bread and $\frac{9}{16}$ lbs. beef.

45. Required the same as in the last example, supposing the rations at first to be 2 lbs. of bread and $1\frac{1}{2}$ lbs. of beef. *Ans.* $1\frac{1}{2}$ lbs. bread and $1\frac{1}{8}$ lbs. beef.

46. If the flooring of a room, 18 feet by 15 feet, cost £6 6s., what will be the cost of a floor 24 feet by 20 feet? *Ans.* £11 4s.

47. Required the cost per square yard of the flooring in the last example. *Ans.* 4s. $2\frac{2}{5}$d.

48. If $50\frac{1}{4}$ yards of calico, 36 inches wide, make 16 shirts, how many yards 27 inches wide will be required to make 24 shirts? *Ans.* $100\frac{1}{2}$.

49. How many yards 2 feet wide would be required? *Ans.* $113\frac{1}{16}$.

50. A field, 400 yards long by 270 wide, was ploughed by 5 horses in 4 days of 9 hours each; how many horses will be required to plough a field 900 yards long by 300 wide in 6 days of 10 hours each? *Ans.* $7\frac{1}{2}$.

No. horses to plough
400 yards by 270 in 4 days of 9 hours = 5.

$$1 \quad ,, \quad 1 \text{ in } 1 \quad ,, \quad 1 \quad ,, \quad = \frac{5 \times 4 \times 9}{400 \times 270}$$

$$900 \quad ,, \quad 300 \text{ in } 6 \quad ,, \quad 10 \quad ,, \quad = \frac{5 \times 4 \times 9 \times 900 \times 300}{400 \times 270 \times 6 \times 10} = 7\tfrac{1}{2}.$$

Or thus. No. square yards in the first field = $400 \times 270 = 108000$;
,, second ,, = $900 \times 300 = 270000$;
and the times are 36 hours and 60 hours respectively. Then
No. horses to plough 108000 sq. yds. in 36 h. = 5.

$$\therefore \quad ,, \quad 270000 \quad ,, \quad 60 \text{ h.} = \frac{5 \times 36 \times 270000}{108000 \times 60} = 7\tfrac{1}{2}.$$

51. How many hours per day should 7 horses work to plough the second field, of the last example, in 6 days? *Ans.* $10\frac{5}{7}$.

52. If either 3 cows or 4 horses can eat up the grass of a field in 90 days, in what time would 2 cows and 6 horses eat up the same?

No. days for 3 cows to eat the grass = 90.

$$\therefore \quad ,, \quad 2 \quad ,, \quad ,, \quad = \frac{90 \times 3}{2} = 135;$$

$\therefore$ Part eaten by 2 cows in 1 day = $\frac{1}{135}$;
and similarly, we find
Part eaten by 6 horses in 1 day = $\frac{1}{60}$.

$\therefore$ Part eaten by 2 cows and 6 horses in 1 day = $\frac{1}{135} + \frac{1}{60} = \frac{13}{540}$;
$\therefore$ No. days for them to eat the whole = $1 \div \frac{13}{540} = 41\frac{7}{13}$.

Or thus. 2 cows will eat as much as twice the 3rd of 4 horses, or $2\frac{2}{3}$ horses; therefore, 2 cows and 6 horses will be equivalent to $8\frac{2}{3}$ horses.

No. days grass for 4 horses = 90.

$$\therefore \quad ,, \quad ,, \quad 8\tfrac{2}{3} \quad ,, \quad = 90 \times 4 \div 8\tfrac{2}{3} = \tfrac{540}{13} = 41\tfrac{7}{13}.$$

53. Required the same as in the last example, supposing 4 cows or 5 horses to eat up the grass in 85 days. *Ans.* 50 days.

54. A policeman starts in pursuit of a boy 160 feet in advance of him. The boy takes 6 steps to the policeman's 5; but 2 of the policeman's

steps are equal to 3 of the boy's. Now supposing the length of the policeman's step to be 2 feet, how far must he run to overtake the boy?

Length of 5 steps of the policeman = 2 ft. × 5 = 10 ft.;
,, 6 ,, ,, boy = $\frac{2}{3}$ of 2 ft. × 6 = 8 ft.;

that is, whilst the policeman moves ever 10 feet, the boy moves over 8 feet, and the space gained by the former = 10 ft. — 8 ft. = 2 ft.; hence we have,

Distance run to gain 2 ft. = 10 ft.
∴ ,, ,, 160 ft. = 5 ft. × 160 = 800 ft.

55. Required the same as in the last example, supposing the boy to take 5 steps for the policeman's 4. *Ans.* 960 ft.

56. A person bought some oranges for 20s. After dividing them into two equal lots, he sold the first lot at a loss of half a farthing each, the second lot at a gain of $\frac{3}{4}$d. each; and received 36s. 8d. for the whole. How many did he buy?

Gain on each = $\frac{1}{2}$ ($\frac{3}{4}$ — $\frac{1}{8}$) = $\frac{5}{16}$d.;
Gain on the whole = 36s. 8d. — 20s. = 16s. 8d.;
∴ No. oranges = 16s. 8d. ÷ $\frac{5}{16}$d. = 200 × $\frac{16}{5}$ = 640.

57. How many did he buy, supposing he sold the first lot at a loss of $\frac{1}{4}$d. each? *Ans.* 800.

58. A person bought spirits at 14s. per gal., and after mixing it with 3 gals. of water, he sold it at 16s. per gal., and thereby gained 68s.; how many gals. of spirits were there?

Gain by the water = 16s. × 3 = 48s.;
∴ Gain on the spirits = 68s. — 48s. = 20s.

but he gained 2s. per gal. on the spirits;

∴ No. gals. spirits = 20s. ÷ 2s. = 10.

59. Supposing he sold the mixture at 14s. per gal., how much water must be added, in order to gain 49s. on the whole? *Ans.* 3$\frac{1}{2}$ gals.

60. A grocer sold 20 lbs. of a package of tea, at a loss of 9d. per lb., and the remainder at a gain of 1s. 3d. per lb., and thereby gained 22$\frac{1}{2}$s. on the whole; how many lbs. were there in the package?

Loss on the 20 lbs. = 9d. × 20 = 15s.;
∴ Gain on the remainder = 22$\frac{1}{2}$s. + 15s. = 37$\frac{1}{2}$s.;

but the gain per lb. on this tea is 1s. 3d. or 1$\frac{1}{4}$s.;

∴ No. lbs. of this tea = 37$\frac{1}{2}$s. ÷ 1$\frac{1}{4}$s. = $\frac{75}{2}$ × $\frac{4}{5}$ = 30;
∴ the wt. of the package = 20 + 30 = 50 lbs.

61. Find the weight of tea in the package, supposing the 20 lbs. to be sold at a gain of 6d. per lb.? *Ans.* 30 lbs.

62. Two railway passengers have together 392 lbs. of luggage, for which they are charged 6s. 3d.; but if the luggage had belonged to one of them, he would have been charged 7s. 6d. How much luggage was each allowed free?

Here, 6s. 3d. = 75d.; and 7s. 6d. = 90d.; then
Charge on the free luggage of 1 person = 90d. — 75d. = 15d.;
∴ Charge on 392 lbs. none of it being free = 90 + 15 = 105d.;
∴ ,, 1 lb. ,, ,, = $\frac{105}{392}$d.;
∴ No. lbs. free luggage to 1 person = 15 ÷ $\frac{105}{392}$ = 56 lbs.

63. A railway passenger with 3 cwt. of luggage was charged 5s. for excess of weight, and another passenger with 4$\frac{1}{2}$ cwt. was charged 8s. 9d. How much luggage was each allowed free?

Here, 5s. = 60d.; 8s. 9d. = 105d.; $4\frac{1}{2}$ cwt. — 3 cwt. = $1\frac{1}{2}$ cwt.;

∴ Charge for $1\frac{1}{2}$ cwt. = 105d. — 60d. = 45d.;

∴ ,, 1 ,, = 45d. ÷ $1\frac{1}{2}$ = 30d.;

∴ No. cwt. in excess in the 1st case = 60d. ÷ 30d. = 2;

∴ Weight allowed free = 3 cwt. — 2 cwt. = 1 cwt.

64. Five gals. of spirit and water cost £3 3s., and 7 gals. containing the same quantity of water cost £4 19s.; how much water was there in each? *Ans.* $1\frac{1}{2}$ gals.

65. A grocer had some tea to sell; if he sold it at 5s. a lb., he would gain 15s., but if he sold it at 4s. 6d. a lb., he would only gain 7s.; how many lbs. were there?

Here, 15s. — 7s. = 8s., and 5s. — 4s. 6d. = 6d.; that is, a difference of 6d. per lb. produces a difference of 8s. in the gain; ∴ the no. lbs. = 8s. ÷ 6d. = 16.

66. A grocer mixed 6 lbs. of inferior tea, at 3s. a lb. with tea at 5s. a lb., and sold the mixture at 5s. 3d. per lb., and thereby gained 23s.; how many lbs. of the latter tea were there?

Gain from the inferior tea = 5s. 3d. × 6 — 3s. × 6 = 13s. 6d.

∴ Gain from the superior tea = 23s. — 13s. 6d. = 9s. 6d.;

but, the gain per lb. on this tea = 5s. 3d. — 5s. = 3d.;

∴ No. lbs. of this tea = 9s. 6d. ÷ 3d. = 114 ÷ 3 = 38.

67. Find the quantity of the latter tea, supposing the mixture to be sold at 5s. 2d., and the gain to be 18s. *Ans.* 30 lbs.

68. From a vessel of wine containing 81 gallons, 27 gallons are drawn off, and the vessel is filled up with water. Find the quantity of wine remaining in the vessel when this has been done five times. *Ans.* $10\frac{2}{3}$ gals.

69. A and B travelled on the same road to London. At the 50th mile stone, A met a coach, travelling at the rate of 10 miles an hour, and $1\frac{1}{2}$ hours afterwards, at the 44th mile stone, he overtook a waggon, travelling at the rate of $2\frac{1}{2}$ miles an hour; B met the coach $\frac{1}{2}$ an hour after A had met it, and overtook the waggon 2 hours after A had overtaken it. At what rates per hour did they travel?

No. miles gone by A in $1\frac{1}{2}$ hours = 50 — 44 = 6;

∴ ,, ,, 1 hour = 6 ÷ $1\frac{1}{2}$ = 4

Dist. B from A when B met the coach = $4 \times \frac{1}{2} + 10 \times \frac{1}{2} = 7$ m.;

,, when B overtook the waggon = $4 \times 2 - 2\frac{1}{2} \times 2 = 3$ m.;

but the interval between these events = $1\frac{1}{2} + 2 - \frac{1}{2} = 3$ hours;

that is, in 3 hours B gains 4 miles on A, ∴ gain per hour = $1\frac{1}{3}$ miles;

∴ B's rate per hour = $4 + 1\frac{1}{3} = 5\frac{1}{3}$ miles.

70. At what distance from London will B overtake A? *Ans.* 27 miles.

PROPORTION.

41. The ratio of one quantity to another is expressed by dividing the former by the latter; thus the ratio of 6 to 8, or 6 : 8, is $\frac{6}{8} = \frac{3}{4}$, that is, 6 is $\frac{3}{4}$ of 8. The first term of a ratio is called the *antecedent* and the second term the *consequent*. Proportion is only another form of expressing an equality of ratios; thus $\frac{3}{4} = \frac{6}{8}$, in the form of a proportion,

is written $3 : 4 :: 6 : 8$, and reads 3 is to 4 as 6 is to 8. The 3 and 6 are called the two antecedents and the 4 and 6 the two consequents. The 3 and 8 are called the *extremes* and the 4 and 6 the *means*.

Again, in the proportion, $3 : 5 :: 6 : 15$, the second term is 5 times the 3rd of the first term, and also the fourth term is 5 times the 3rd of the third term. Hence the first three terms of a proportion being given the fourth term may be readily found as follows,

$$7 : 9 :: 28 : \text{what number?}$$

Here the 2nd term is $\frac{9}{7}$ of the 1st; therefore, the 4th term will be $\frac{9}{7}$ of the 3rd, that is, the 4th term reqnired $= \frac{9}{7}$ of $28 = \frac{9 \times 28}{7} = 36$. Here it will be observed, that *the 4th term is found by multiplying together the 2nd and 3rd and dividing the product by the 1st*, that is, in any proportiòn

$$\text{the 4th term} = \frac{\text{2nd term} \times \text{3rd term}}{\text{1st term}} \dots (1).$$

The Rule of Proportion, as applied to commercial arithmetic, is based on this principle, as we shall afterwards see.

Multiplying each side of equality (1) by the "1st term," we get

$$\text{1st term} \times \text{4th term} = \text{2nd term} \times \text{3rd term}$$

that is, *the product of the extremes is equal to the product of the means.*

In every proportion the 1st and 2nd terms must be of same kind or name, and also the 3rd and 4th; but the two former terms need not have the same name as the two latter. Thus we have, 6 lbs. : 2 lbs. :: 12s. : 4s.; where units of lbs. are compared in the 1st and 2nd terms, and units of shillings in the 3rd and 4th; so that the ratio of 6 lbs. : 2 lbs. is the same as 12s. : 4s.

In applying proportion to the solution of questions in Rule of Three, we must first assure ourselves that the quantities are strictly in proportion, and if so, whether they are in *direct* proportion or in *inverse* proportion. For illustration let us take Ex. 4, Art. 6.

Here to be assured that this is a case of *direct* proportion, we observe that the cost must be in the ratio of the weight, that is, if we *double* the weight the cost will also be *double*, and so on; then

36 oz. : 23 oz. :: Cost of 36 oz. : Cost of 23 oz.; that is,
36 : 23 :: 15s. : Cost of 23 oz.

Then by the rule, expressed in eq. (1), we obtain the *Cost of* 23 *oz.* by multiplying 15s. by 23 and dividing the product by 36.

As another illustration let us take Ex. 82, Art. 39.

$$\begin{array}{r} 5 \text{ days} : 6\frac{1}{2} \text{ days} :: 8 \text{ hours} \\ 6\frac{1}{2} \\ \hline 5\,)\,52 \\ \hline 10\frac{2}{5} \text{ hours} \end{array}$$

Here the term $6\frac{1}{2}$ days corresponds to the term 8 hours; if we *double* the No. of days, the No. hours required will be only *half* of what it was before; so that the No. days varies *inversely* as the No. hours per day required. The 3rd term is 8 hours, because it has the same name as the answer, and we put the *corresponding* term, $6\frac{1}{2}$ days, *second* instead of *first*, as in the case of *direct* proportion.* Then by the rule, we multiply 8 by $6\frac{1}{2}$ and divide by 5, for the No. hours required.

---

* Given nearly in the language of Colenso.

Proportion is an elegant mode of expressing the results of arithmetical investigations, and forms an essential feature of many departments of mathematics; but considered as *a rule* of commercial arithmetic, it is neither simple nor expeditious. It substitutes special rules for simple, and therefore obvious, processes of reasoning, and in cases of compound proportion, it gives a repulsive abstraction to the forms of solution. By *the equational method*,* all questions, of this kind, may be solved by multiplication and division. In the solution of a question by the rule of proportion, the mental process, whether expressed or understood, by which we determine whether the question comes under the rule of direct or of indirect proportion, really gives the steps which should be taken in solving the proposed question by the equational method.

SIMPLE INTEREST.

42. *Interest* is the sum paid for the loan of money. The sum of money lent is called *the principal;* and *the amount* is the sum of the principal and interest. The *rate per cent.* is the interest or sum paid for the loan of £100 for 1 year or per annum.

1. Required the interest of £13 for 1 year at $2\frac{1}{3}$ per cent.

$$\text{Interest on } £100 = £2\tfrac{1}{3}$$

$$\therefore \quad " \quad £1 = £\frac{2\frac{1}{3}}{100}$$

$$\therefore \quad " \quad £13 = £\frac{2\frac{1}{3} \times 13}{100}\,*$$

$$= £\frac{7 \times 13}{300} = £\cdot3033 = 6\text{s. } 0\tfrac{3}{4}\text{d.}$$

* Here *the interest is found by multiplying the principal by the rate and dividing by* 100.

2. Required the interest of; £18 at $3\frac{1}{2}$ per cent.; £28 at $4\frac{1}{4}$ per cent. *Ans.* 12s. 7d.; £1 3s. $9\frac{1}{2}$d.

3. What is the interest of $£9\frac{1}{2}$ for 3 years, at $4\frac{1}{4}$ per cent. per annum?

$$\text{Interest on } £100 \text{ for 1 y.} = £4\tfrac{1}{4}$$

$$\therefore \quad " \quad £1 \;"\; 1 \text{ y.} = £\frac{4\frac{1}{4}}{100}$$

$$\therefore \quad " \quad £1 \;"\; 3 \text{ y.} = £\frac{4\frac{1}{4} \times 3}{100}$$

$$\therefore \quad " \quad £9\tfrac{1}{2} \;"\; 3 \text{ y.} = £\frac{4\frac{1}{4} \times 3 \times 9\frac{1}{2}}{100}\,\dagger$$

$$= £\frac{17 \times 3 \times 19}{800} = £1\cdot21125$$

$$= £1 \text{ 4s. } 2\tfrac{1}{2}\text{d.}$$

† Here *to find the interest we multiply the principal, the rate, and the number of years together, and divide by* 100.

---

* First given by the Author as a *general method* in his "Principles of Arithmetic," published in 1842.

4. Required the interest of; £75 for 3 years, at 4 per cent. per annum; £25 for 2 years, at 4½ per cent. per annum. *Ans.* £9; £2 5s.

5. Find the amount of £58 19s. 2d. for 2⅔ years at 3 per cent.

	£	s.	d.	
	58	19	2	
			8	= 2⅔ × 3
100)	4|71	13	4	
	20			
14	33			
	12			
4	00			

Here to find the interest we multiply the principal by the product of the rate and number of years, and divide by 100. We divide by 100 by cutting off two figures of the dividend.

	£	s.	d.
Interest =	4	14	4
Principal =	58	19	2
∴ Amount =	63	13	6

6. Find the amount of; £139 6s. 3d. for 4 years, at 2½ per cent.; £36 10s. for 3 years, at 4 per cent.; £57 6s. 8d. for 7 years, at 2½ per cent. *Ans.* £153 4s. 10½d.; £40 17s. 7d.; £67 7s. 4d.

7. Required the interest of; £24 9s. 4d. at 4⅛ p. c.; £6·15 at 2⅔ p. c.

	£	s.	d.
	24	9	4
			4⅛
	97	17	4
For ⅛	3	1	2
1,00)	1|00	18	6
	20		
0	18		
	12		
2	22		

Ans. £1 0s. 2d.

	£
	6·15
	2⅔
	12·30
For ⅔	4·10
1,00)	·164 £
	20
	3·280s.
	12
	3·36d.
	4
	1·44

Ans. 3s. 3¼d.

8. Required the interest of; £9 13s. 4d. at 3¾ p. c.; £14 10s. 5d. at 3⅓ p. c.; £61·5 at 2½ p. c. *Ans.* 7s. 3d.; 9s. 8d.; £1 10s. 9d.

9. What is the interest of £60 3s. for 7 months, at 4 p. c. per annum?

$$\text{Interest for 12 mon.} = \frac{£60\ 3s. \times 4}{100}$$

$$\text{,, } \quad 1 \text{ ,, } = \frac{£60\ 3s. \times 4}{12 \times 100}$$

$$\text{,, } \quad 7 \text{ ,, } = \frac{£60\ 3s. \times 4 \times 7}{12 \times 100} = £1\ 8s.\ 0\tfrac{3}{4}d.$$

Or thus. Calculating the interest for 12 months or 1 year, we have,

		£	s.	d.
	Interest for 1 y. =	2	8	$1\frac{1}{2}$
6 mon. = $\frac{1}{2}$ y.	„ 6 m. =	1	4	$0\frac{3}{4}$
1 mon. = $\frac{1}{6}$ of 6 mon.	„ 1 m. =		4	0
∴	„ 7 m. =	1	8	$0\frac{3}{4}$

10. Required the interest of; £23 5s. 10d. for 4 mon., at 5 p. c.; £650 15s. for 3 mon., at $1\frac{1}{4}$ p. c. *Ans.* 7s. 9d.; £2 0s. 8d.

11. Required the interest of £89 4s., from May 8 till July 23, at 3 per cent. per annum.

Here the number of days from the 8th of May to the 23rd of July = 76.

$$\text{Interest for 365d.} = \frac{£89\ 4s. \times 3}{100}$$

$$\therefore \quad ,, \quad 76d. = \frac{£89\ 4s. \times 3 \times 76}{36500}$$

$$= \frac{£89\ 4s. \times 6 \times 76}{73000} = 11s.\ 1\tfrac{3}{4}d.$$

Hence the following rule: *Rule.—To find the interest for any number of days, multiply the principal by twice the rate and the product by the days, and divide the result by* 73000.

12. Required the interest of; £349 7s. 4d. for 178 days, at $2\frac{1}{2}$ per cent.; £762 3s. 4d. for 132 days, at 3 per cent.

Ans. £4 5s. $2\frac{1}{4}$d.; £8 5s. $4\frac{1}{2}$d.

Simplified forms of calculation.

43. In finding the interest for any number of days, the division by 73000 may be simplified by the following rule: *Rule.*—Take one-third the dividend (rejecting shillings and pence), one-tenth of that third, and one-tenth of that tenth; then add the four lines together, cut off five figures from this sum, and in the result reject a farthing for each £10.

1. Required the interest of £873 10s. for 128 days, at 4 p. c.

By the ordinary rule.

```
     £873  10
             8
     ---------
      6988   0
       128
     -----
     55904
    83856
    ------
73000)894464
    ------
 £12  5  0½.  Ans.
```

By the rule.

```
              894464
         ⅓.   298154
               29815
                2981
          ----------
          £12·25414
          ----------
           12   5  0¾
Correction         0¼
          ----------
          £12   5  0½
```

2. Required the interest of £73 6s. for 84 days, at 4½ p. c.

Ans. 15s. 2d.

Proof of the rule. The true quotient $= \dfrac{\text{Dividend}}{73000}$; and by the rule the quotient $= \dfrac{\text{Dividend}}{100000}\left(1 + \frac{1}{3} + \frac{1}{30} + \frac{1}{300}\right) = \dfrac{\text{Dividend}}{73000} \times 1{\cdot}0001$, where the result is $\frac{1}{10000}$ in excess of the true quotient. Now in 10000 farthings the error would be one farthing but 10000 farthings = £10 8s. 4d.

44. When the rate per cent. is 5, or some simple aliquot part of 5, the calculation may be much simplified by the following methods.

It has been shown (Ex. 33, Art. 6) that the interest on £1 at 5 p. c. is 1s., therefore the interest on 10s. = 6d., int. on 5s. = 3d., int. on 2s. 6d. = 1½d., on 1s. 3d. = ¾d., and on 5d. = ¼d. Whence the interest on any sum at 5 p. c. may be easily calculated.

1. Required the interest of £68 11s. 10d. at 5 p. c.

Interest on £68 = 68s. =	£3	8	0
,, 10s. . . . . =			6
,, 1s. 8d. = 5d. × 4 =			1
∴ ,, £68 11s. 8d. =	£3	8	7

which is the interest true to a farthing. It is scarcely necessary to observe that the answer may be *at once* written down.

2. Required the interest of £160 7s. 8d. for 4 years, at 5 p. c.

£	s.	d.
160	7	8
		4
641	10	8
32	1	6¼ *Ans.*

Here, in order to avoid errors arising from the neglecting of remainders, we first multiply by the number of years, and then the interest on this result, as a principal, may be written down at once, giving the answer true to a farthing.

3. Find the interest of the following sums at 5 p. c.; £7 6s. 7d.; £319 3s.; £406 16s. 5d. for 6 years.

Ans. 7s. 3¾d.; £15 19s. 1¾d.; £122 0s. 11d.

45. Having found the interest of any sum at 5 p. c., the interest at any other rate may be calculated by aliquot parts.

1. Required the interest of £164 10s. 11d. at 3¾ p. c.

	Int. at 5 p. c. =	£8	4	6½
2½ = ½ of 5	,, 2½ ,, =	4	2	3¼
1¼ = ½ of 2½	,, 1¼ ,, =	2	1	1⅝
∴	,, 3¾ ,, =	6	3	4¾

2. Required the interest of £226 7s. 9d. at 4½ p. c.

	Int. at 5 p. c. =	£11	6	4½
½ = $\frac{1}{10}$ of 5	„ ½ „ =	1	2	7½
	„ 4½ „ =	10	3	9

The results may be carried out to the fractions of farthings if required. When there are a given number of years, first multiply the principal by the number of years, as in example 2, Art. 44.

3. Find the interest of the following sums; £780 15s. 10d. at 4 p. c.; £64 3s. 1½d. for 4 years at 6¼ p. c. *Ans.* £31 4s. 7½d.; £16 0s. 9¼d.

4. Find the interest of £64 3s. 4½d. at 10 p. c. *Ans.* £6 8s. 4d.

Here we might find the interest of 5 p. c., and then double it for the interest at 10 p. c., but the interest may be also found as follows. As 10 is the tenth of 100, the interest required will be $\frac{1}{10}$ of the principal.

Having found the interest of any sum at 10 p. c., the interest at any other rate may be calculated by aliquot parts.

5. Required the interest of £40 15s. at 9 p. c.

	10)	£40	15	0
	Int. at 10 p. c. =	4	1	6
1 = $\frac{1}{10}$ of 10	„ 1 „ =		8	1¾
∴	„ 9 „ =	3	13	4

6. Find the interest of; £71 5s. 2½d., at 8 p. c.; £31 11s. 3d. at 7½ p. c.; £61 11s. 8d. at 2 p. c. *Ans.* £5 14s.; £2 7s. 4d.; £1 4s. 7½d.

7. Find the interest of £16 11s. 2d. for ½ year, at the following rates per cent.; at 2½; at 1¼; at 4; at 6; at 3½; at 1; at 5⅝.

Ans. 4s. 1¼d.; 2s. 0¾d.; 6s. 7¼d.; 9s. 11d.; 5s. 9½d.; 1s. 7¾d.; 9s. 7¾d.

46. *Exercises.* Find the interest of the following sums at the given rates per cent. per annum, &c.

	£	s.	d.			£	s.	d.
1.	188	6	4	at 4 p. c.	*Ans.*	7	10	7¾
2.	175	8	2	at 3 p. c.	„	5	5	2¾
3.	205	10	11	at 5½ p. c.	„	11	6	1
4.	1308	13	7½	at 3½ p. c.	„	45	16	0¾
5.	294	16	8	at 3¾ p. c.	„	11	1	1½
6.	19	18	0½	at 5⅛ p. c.	„	1	0	4¾
7.	449	15	3¾	at 2⅜ p. c.	„	10	13	7¾
8.	1275	2	9	at 5⅚ p. c.	„	74	7	7¾
9.	49	11	2	for 4 years, at 2¾ p. c.	„	5	9	0¼
10.	324	10	0	for 3 mon., at 2½ p. c.	„	2	0	6¾
11.	246	18	4	for 3 y. 4 mon., at 3 p. c.	„	24	13	10
12.	89	16	5½	for 1 y. 7 mon., at 5 p. c.	„	7	2	2½
13.	292	9	4	for 1 y. 9 mon., at 3⅛ p. c.	„	15	19	10½
14.	228	5	0	for 31 days, at 2½ p. c.	„	0	9	8¼
15.	394	7	5	for 98 days, at 6½ p. c.	„	6	17	7¾
16.	304	3	4	for 29 days, at 4 p. c.	„	0	19	4

	£	s.	d.			£	s.	d.
17.	455	7	6	for 54 days, at $2\frac{1}{4}$ p. c.	*Ans.*	1	13	8
18.	425	0	0	for 154 days, at $4\frac{1}{2}$ p. c.	,,	8	1	$4\frac{1}{2}$
19.	1602	7	0	from May 3 to June 9, at 5 p. c.	,,	8	2	5
20.	390	7	$4\frac{3}{4}$	from June 20 to July 26, at 4 p. c.	,,	1	10	$9\frac{1}{2}$
21.	455	7	6	from Sept. 2 to Nov. 9, at $4\frac{1}{2}$ p. c.	,,	3	16	4

47. *To find the interest on two or more sums at the same rate, but for different times.*

1. Required the total interest on the following sums at 5 p. c.; £874 for 17 days; £437 for 68 days.

Here we might find the interest for each sum, and then add the results; but the following method is more expeditious,

$$\text{Int. on £874} = \frac{874 \times 17 \times 10}{73000}; \text{ Int. on £437} = \frac{437 \times 68 \times 10}{73000};$$

$\therefore$ Total Int. $= (874 \times 17 + 437 \times 68) \times \frac{10}{73000} = £6\ 2s.\ 1\frac{1}{2}d.$

Hence we have the following rule: *Rule.*—Multiply each sum by the number of days on which interest is due, add the products, multiply this sum by double the rate and divide 73000.

2. Required the total interest on £32 17s. for 80 days, £25 11s. for 40 days, at 4 per cent. *Ans.* 8s.

3. Required the interest on a bill of £228 5s. due June 3, of which £28 5s. were paid July 4, and the remainder Aug. 23, at 5 p. c. *Ans.* £2 6s. $9\frac{1}{4}$d.

48. *To find the interest on Accounts Current.*

An Account Current is a Dr. and Cr. statement of the mercantile transactions between two parties. The left hand side of the account, or Dr., contains the claims of the merchant who furnishes it, and the other side, or Cr., contains the payments made to him.

1. Required the interest, &c., on the following account till Dec. 31, at 5 p. c.

Dr. Mr. May, in Account Current, with R. Day. Cr.

	£	s.	d.		£	s.	d.
Jan. 11, To Goods	500	10	0	Ap. 10, By Cash	400	0	0
Feb. 18, To Goods	260	5	0	June 20, By a Bill	600	0	0
May 20, To Goods	450	6	8	July 21, By flour	60	0	0
Dec. 31, To balance of interest	17	10	$10\frac{1}{4}$	Dec. 31, By balance due R. Day	168	12	$6\frac{1}{4}$
	1228	12	$6\frac{1}{4}$		1228	12	$6\frac{1}{4}$

Here £500 10s. will be chargable with interest from Jan. 11 to Dec. 31, that is, for 354 days, and so on to the other sums. Hence we have by the rule Art. 43,

£	s.	d.		£
500	10	0	× 354 =	177177
260	5	0	× 316 =	82239
450	2	6	× 224 =	100828
				360244
				232180
			Difference	128064

£		£
400	× 265 =	106000
600	× 194 =	116400
60	× 163 =	9780
		232180

$\therefore$ Interest $= 128064 \times \frac{10}{73000} =$ £17 10s. $10\frac{1}{4}$d.

2. Required the interest, &c., on the following account till Dec. 20, at 4 per cent. per annum.

Dr.	£	s.	d.
Feb. 20, To balance acct.	93	3	9
Mar. 7, To Goods	107	5	0
June 30, To Goods	207	14	2
Dec. 20, To balance of Int.	2	9	$8\frac{1}{4}$
	410	12	$7\frac{1}{4}$

Cr.	£	s.	d.
Ap. 3, By Cash	83	6	8
„ 16, By Cash	173	19	0
Oct. 6, By a bill	100	0	0
Dec. 20, By balance	53	6	$11\frac{1}{4}$
	410	12	$7\frac{1}{4}$

49. *To find the rate when the principal and interest are given.*

1. If the interest on £220 is £9 7s., what is the rate per cent.?

Int. on £220 = £9 7s.

$\therefore$ „ £100 $= \frac{£9\ 7s. \times 100}{220} =$ £4 5s., or $4\frac{1}{4}$.

2. If the interest on £55 is £2 4s., what is the rate per cent.? *Ans.* 4.

3. At what rate per cent. per annum, simple interest, will £750 amount to £925, in $4\frac{1}{2}$ years?

Int. on £750 for $4\frac{1}{2}$ y. = £175

$\therefore$ „ £100 „ 1 y. $= £\frac{175 \times 100}{750 \times 4\frac{1}{2}} = £5\frac{5}{27}$, or £5 3s. $8\frac{4}{9}$d.

4. At what rate per cent. per annum, simple interest, will £450 amount to £490 10s., in 2 years? *Ans.* $4\frac{1}{2}$.

5. With a capital of £4000, a merchant gained £650 in $1\frac{1}{2}$ years; what was his gain per cent. per annum? *Ans.* $10\frac{5}{6}$.

50. *To find the principal when the interest and rate are given.*

1. What principal will bring an interest of £24 in 60 days, at $4\frac{1}{2}$ p. c.? Here as £100 gives £$4\frac{1}{2}$ interest in 1 year or 365 days, we write

Principal to give £$4\frac{1}{2}$ in 365 d. = £100

$\therefore$ „ „ £24 in 60 d. $= £\frac{100 \times 24 \times 365}{4\frac{1}{2} \times 60}$

= £3244 8s. $10\frac{2}{3}$d.

2. What principal will bring £40 interest in 146 days at 5 per cent. per annum? *Ans.* £2000.

3. What principal, at 5 per cent., will bring a yearly income of £128? *Ans.* £2560.

51. *To find the time when the principal, rate, and amount are given.*

1. In what time will £500 amount to £516, at 4 p. c. per annum?

Here the interest of £500 for 1 year is £20, that is,

$$\text{Time to produce £20 Int.} = 1 \text{ y.}$$

$$\therefore \quad ,, \quad ,, \quad £16 \; ,, = \frac{1 \times 16}{20} \text{ y.}$$

$$= \frac{365 \times 16}{20} \text{ d.} = 292 \text{ days.}$$

Hence we have the following rule. *Rule.*—Divide the difference between the principal and amount by the interest of the principal for 1 year, and the quotient will be the number of years, or parts of a year, required.

2. In what time will £800 amount to £850, at 5 per cent. per annum? *Ans.* 1 y. 3 mon.

3. In what time will £230 amount to £250, at 9 per cent. per annum? *Ans.* 352·657 days.

BANKERS' DISCOUNT.

52. *Discount* is the abatement which must be made for the *present payment* of a debt or bill which is not due until a certain time hence. The Bankers' discount is the interest of the bill at the given rate and for the time which it has to run, reckoning from the time at which it is discounted till it is due, including 3 days of grace.

1. What is the discount and present worth of a bill of £109 10s., drawn or dated April 20, at 3 months, and discounted on May 24, at 5 per cent. per annum? *Ans.* 18s.; £108 12s.

Here the bill is due on July 20, but allowing 3 days of grace on July 23. Now from May 24 to July 23 is 60 days, which is the time the bill has to run. By the rule Art. 43, we find the interest, or discount, of £109 10s. for 60 days at 5 p. c. to be 18s. Hence the present payment or worth = £109 10s. — 18s. = £108 12s.

2. Required the present worth of a bill of £290, drawn April 15, at 4 months, and discounted on May 20, at 5 p. c. *Ans.* £286 8s. $5\frac{3}{4}$d.

53. *Exercises.* Find the present worths of the following bills, at the given rates per cent. per annum:

	£	s.	d.	Drawn			Discounted			£	s.	d.
1.	208	1	8,	Feb. 28,	at 7	mon.,	June 8,	at 4	*Ans.*	205	8	$3\frac{1}{2}$
2.	266	13	4,	Sep. 10,	,, 5	,,	Nov. 17,	,, $4\frac{1}{2}$	,,	263	15	$5\frac{1}{2}$

	£	s.	d.	Drawn		Discounted		£	s.	d.
3.	303	11	8,	June 1,	at 5 mon.,	July 20, at $5\frac{1}{2}$	*Ans.*	298	13	$9\frac{1}{2}$
4.	447	16	0,	Jan. 8,	,, 11 ,,	May 12, ,, 5	,,	434	14	$8\frac{1}{4}$
5.	243	9	4,	Ap. 1,	,, 10 ,,	June 26, ,, 5	,,	236	0	7
6.	284	6	$4\frac{1}{2}$,	May 1,	,, 7 ,,	June 7, ,, 5	,,	277	6	2

TRUE DISCOUNT.

54. The Bankers' discount is always in excess of the true discount; for the true discount is obviously the interest of the *present worth* of the bill for the given time, and not the interest of the amount of the bill itself.

1. What is the *true* discount and present worth of a bill of £714 due at 4 months hence, at 6 per cent. per annum?

Here the interest of £100 for 4 months is £2, therefore £100 paid at the present time will be equivalent to £102 paid 4 months hence,

$$\therefore \text{ Discount on } £102 = £2$$

$$\therefore \quad ,, \quad £1 = £\frac{2}{102}$$

$$\therefore \quad ,, \quad £714 = £\frac{2 \times 714}{102} = £14$$

$$\therefore \text{ Present worth} = £714 - £14 = £700.$$

In this case, the bankers' discount will be £14 5s. 7d., that is, 5s. 7d. in excess of the true discount.

Or thus. Present worth £102 = £100

$$\therefore \quad ,, \quad £714 = £\frac{100 \times 714}{102} = £700$$

$$\therefore \text{ Discount} = £714 - £700 = £14, \text{ as before.}$$

2. What is the true discount and present worth of a bill of £78, due at 1 year hence at 5 per cent.? *Ans.* £3 14s. $3\frac{1}{2}$d.; £74 5s. $8\frac{1}{2}$d.

3. Required the present worth of a bill of £156, due at 1 year hence at 4 per cent. *Ans.* £150.

4. Required the present worth of £250, due 9 months hence, at 4 per cent. per annum. *Ans.* £242 14s. $4\frac{1}{2}$d.

5. Required the present worth of £64 10s., due 3 months hence, at 5 per cent. per annum.

Here the interest of £100 for 3 months is £1 5s.

$$\therefore \text{ Present worth } £101\ 5s., \text{ or } 2025s. = £100$$

$$\therefore \quad ,, \quad £64\ 10s., \text{ or } 1290s. = £\frac{100 \times 1290}{2025}$$

$$= £63\ 14s.\ 0\tfrac{3}{4}d.$$

6. Required the present worth of £350, due 511 days hence, at 3 p. c. per annum.

The interest of £100 for 511 days is £4$\frac{1}{5}$.

$\therefore$ Present worth £104$\frac{1}{5}$ = £100

$$\therefore \quad " \quad £350 = £\frac{100 \times 350}{104\frac{1}{5}} = £335\ 17s.\ 10d.$$

Here £104$\frac{1}{5}$ is the amount of £100 for 511 days at 3 p. c.; hence we derive the following rule:

Rule I.—To find the present worth of a given sum, multiply the given sum by 100 and divide by the amount of £100 for the given time.

When the time is expressed in days, the following rule is best adapted for calculation:

Rule II.—Multiply the given sum by 36500, and divide the product by 36500 increased by the product of the days and rate.

Proof. In the foregoing example, the interest of £100 for the given time is expressed by £$\frac{511 \times 3}{365}$.

$$\therefore \text{ Present worth of } £(100 + \frac{511 \times 3}{365}) = £100$$

$$\therefore \quad " \quad £350 = £\frac{100 \times 350}{100 + \frac{511 \times 3}{365}} = \frac{36500 \times £350}{36500 + 511 \times 3}$$

7. Required the present worth of £516, due 292 days hence, at 4 per cent. per annum. *Ans.* £500.

8. What sum lent at 5 per cent. for a year would produce £630?

Sum to produce £105 = £100

$$\therefore \quad " \quad £630 = £\frac{100 \times 630}{105} = £600.$$

9. What sum lent at 4 per cent. for a year would produce £520? *Ans.* £500.

10. What is the present worth of £545 10s., payable, £25 10s. at 6 months, and the remainder at 12 months, at 4 p. c. discount?

$$\text{Present worth } £25\ 10s. = \frac{100 \times £25\ 10s.}{102} = £25$$

$$\therefore \quad " \quad £520 = \frac{100 \times £520}{104} = £500$$

$$\therefore \quad " \quad £545\ 10s. = £25 + £500 = £525.$$

11. What is the present worth of £300, payable, £100 at 4 months, £100 at 8 months, and the remainder at 12 months, at 5 per cent. discount? *Ans.* £290 7s. 5$\frac{1}{4}$d.

12. Required the present worth of a bill of £164 4s., drawn May 1, at 3 months, and discounted May 23, at 6 per cent. per annum.

Here the bill would be due on Aug. 1, but allowing 3 days of grace it is due on Aug. 4. From May 23 to Aug. 4 is 73 days, which is the time the bill has to run; hence we have by Rule II.—

$$\text{Present worth} = \frac{36500 \times £164\ 4s.}{36500 + 73 \times 6} = £162\ 5s.\ 0\tfrac{3}{4}d.$$

55. *Exercises.* Find the true present worths of the bills of the exercises Art. 53, page 69.

Answers.

	£	s.	d.			£	s.	d.
1.	205	8	11¾		4.	435	2	1
2.	263	16	1		5.	236	4	11¾
3.	298	15	3¾		6.	277	9	6½

COMMISSION, INSURANCE, BROKERAGE, ETC.

56. *Commission* is an allowance of so much per cent. to an Agent for buying or selling of goods, or any other property. *Brokerage* is a similar allowance to a Broker for negociating the sale of stock, or bills, &c. *Insurance* is a contract in which the Underwriter of a company engages to repay losses that may be sustained by the insured, in consideration of a certain allowance per cent., called the premium.

To find the allowance of commission, insurance, or brokerage, as the case may be, on a given sum at a given rate per cent., *multiply the sum by the rate and divide by* 100. The calculation is often simplified by the method of aliquot parts, as in the following example.

1. Required the premium of insurance on £320 4s. 6d., at £4 5s. 10d. per cent.

Here, the multiplication of the sum by the rate is performed by multiplying by 4 and taking parts for 5s. 10d.; the product thus obtained divided by 100 gives the premium required.

	£	s.	d.
	320	4	6
			4
	1280	18	0
5s. = ¼	80	1	1½
10d. = ⅙	13	6	10¼
100)	1374	5	11¾
	13	14	10¼

2. Required the premium of insurance on £256 4s. 8d., at £3 8s. 3d. per cent. *Ans.* £8 14s. 10½d.

3. Required the commission on £882 8s. 3d., at 3½ per cent. *Ans.* £30 17s. 8¼d.

4. Required the brokerage on £420, at ½ p. c. *Ans.* £2 2s.

5. What is the brokerage on £473 9s. 5d., at 5s. 6d. p. c.? *Ans.* £1 6s. 0½d.

6. If an Agent gets 1s. 6d. in the pound for collecting rents, how much does he get per cent.?

Commission on £1 = 1s. 6d.

∴ „ £100 = 1s. 6d. × 100 = £7½.

7. Two pence in the shilling is how much per cent.? *Ans.* 16⅔.

8. What sum must be insured, at 8½ per cent. on goods worth £300, so that, in the case of loss, the owner may be repaid the value of the goods as well as the premium of insurance?

Here £100 insured, in the case of loss, will give the insurer £100 — £8½ or £91½; hence we write,

$$\text{Sum insured to give } £91\tfrac{1}{2} = £100$$

$$\therefore \quad ,, \quad £300 = \frac{£100 \times 300}{91\tfrac{1}{2}} = £327\ 17\ 4\tfrac{1}{2}\text{d.}$$

Here, we *multiply the value of the goods by* 100, *and divide by* 100, *less by the rate.*

9. Required the same as in the last example, when the rate is $10\frac{1}{2}$, and the value of the property is £1790. *Ans* £2000.

10. Required the same as in Ex. 8, when the rate is $5\frac{3}{4}$, and the value of the goods is £969 6s. 3d. *Ans.* £1028 8s. $11\frac{1}{2}$d.

11. Required the premium on a policy of £450 at £2 4s. per cent. *Ans.* £9 18s.

12. Bought a property for £520, with a gross annual rental of £35, out of which I paid 4 per cent. to the house agent for commission, and £3 for rates; what per centage did I receive on my money?

$$\text{Deduction for commission and rates} = \frac{35 \times 4}{100} + 3 = £4{\cdot}4,$$

$$\therefore \text{Net rental} = £35 - £4{\cdot}4 = £30{\cdot}6,$$

$$\therefore \text{Int. on } £520 = £30{\cdot}6, \therefore \text{Int. on } £100 = £\frac{30{\cdot}6 \times 100}{520} = £5\ 17\text{s. } 8\tfrac{4}{13}\text{d.}$$

13. Required the same as in the last example, supposing the charge for commission to be 5 per cent. and the rates £6. *Ans.* £5 4s. $9\frac{3}{13}$d.

BUYING AND SELLING STOCKS.

57. STOCK is the debt owing by Government, called the Public Funds, or it is the capital of a bank or any trading company. The price of stock varies with the state of the money market, &c. When the market price of £100 stock is £100 the funds or shares, as the case may be, are said to be at *par*, when the price is less than £100, they are said to be at a *discount*, and when the price is greater than £100, they are said to be at a *premium* which very seldom happens with the funds. Good railway shares, and other kinds of securities, giving a high per centage, are generally at a premium.

1. What would a person have to pay for £250 stock, at $93\frac{1}{4}$ per cent., the charge for brokerage being $\frac{1}{8}$ per cent.?

$$\text{Cost } £100 \text{ stock} = £93\tfrac{1}{4} + £\tfrac{1}{8} = £93\tfrac{3}{8}$$

$$\therefore \ ,, \ £250 \ ,, \ = \frac{£93\frac{3}{8} \times 250}{100} = £233\ 8\text{s. } 9\text{d.}$$

Here, to find the cost of stock, we multiply the amount of stock by the price per cent. added to that of the brokerage, and divide by 100.

It will be observed that it is customary to charge the brokerage on the *nominal* value of the stock; whereas, according to mathematical principles, it should be charged upon the money actually paid for the stock. When the stock is at a discount, the usual mode of calculation gives an advantage to the broker, and on the contrary, when the stock is at a premium the advantage is in favor of the purchaser. For example, the purchase of £100 stock, at £50 per cent. would give £$\frac{1}{8}$ to the broker, according to the usual mode, whereas if the charge is taken on the £50 it would only give £$\frac{1}{16}$ brokerage.

2. Required the respective costs of the following amounts of stock, at the given prices per cent., the brokerage in each case being $\frac{1}{8}$ per cent.; £400 at $72\frac{5}{8}$; £375 at $90\frac{1}{2}$; £625 at $79\frac{1}{4}$; £250 at $90\frac{3}{8}$; £150 at $93\frac{1}{3}$; £730 at $62\frac{5}{6}$.

Ans. £291; £339 16s. $10\frac{1}{2}$d.; £496 1s. $10\frac{1}{2}$d.; £226 5s.; £140 3s. 9d.; £459 11s. 11d.

3. What is the value of £625 of Bank Stock at $175\frac{1}{2}$ per cent.? *Ans.* £1096 17s. 6d.

4. What quantity of stock (neglecting the charge of brokerage) can be purchased for £1200, at 184 per cent.?

Amount of stock for £184 = £100

$$\therefore \quad " \quad " \quad £1200 = £\frac{100 \times 1200}{184} = £652 \text{ 3s. } 5\tfrac{3}{4}\text{d.}$$

5. Required the quantities of stock which can be purchased at the given prices per cent., as in the last example, for the following sums: for £360 at 90; for £5166 at $215\frac{1}{4}$; for £170 at $73\frac{1}{4}$; for £1200 at 159; for £600 at $61\frac{3}{4}$; for £780 at $89\frac{1}{4}$.

Ans. £400; £2400; £232 1s. $7\frac{1}{2}$d.; £754 14s. 4d.; £971 13s. $2\frac{1}{4}$d.; £873 18s. $11\frac{3}{4}$d.

6. What quantity of 3 per cent. Consols will £780 purchase at $97\frac{3}{8}$ per cent., brokerage being $\frac{1}{8}$ per cent. charged on stock purchased?

In this case, the cost per cent. $= 97\frac{3}{8} + \frac{1}{8} = 97\frac{1}{2}$; hence we write,

Amount of stock for £$97\frac{1}{2}$ = £100

$$\therefore \quad " \quad " \quad £780 = \frac{100 \times 780}{97\frac{1}{2}} = £800.$$

7. What quantity of stock will £291 purchase at $72\frac{5}{8}$ per cent., brokerage being $\frac{1}{8}$ per cent.? *Ans.* £400.

8. What rate per cent. interest will be got by buying 3 per cent. stock at 72 per cent.?

Here as £72 is paid for £100 stock we have

Int. on £72 = £3

$$\therefore \quad " \quad £100 = £\frac{3 \times 100}{72} = £4\tfrac{1}{6}.$$

9. What rate per cent. interest will be got by buying 5 per cent. railway stock, at 112 per cent.? *Ans.* $4\frac{13}{28}$.

10. What rate per cent. interest will be obtained by buying Bank Stock at 91 per cent., the dividend being $6\frac{1}{2}$ per cent.? *Ans.* $7\frac{1}{7}$.

11. A person bought £100 Bank Stock, giving 8 per cent. dividend, for £160, besides paying $\frac{1}{8}$ per cent on the purchase money for brokerage. What rate per cent. interest will he get for his money?

$$\text{Cost } £100 \text{ stock} = £160 + £\frac{160 \times \frac{1}{8}}{100} = £160\tfrac{1}{5}, \text{ then}$$

Int. on £$160\frac{1}{5}$ = £8

$$\therefore \quad " \quad £100 = £\frac{8 \times 100}{160\frac{1}{5}} = £4 \text{ 19s. } 10\tfrac{1}{2}\text{d.}$$

12. Required the same as in the last example, when £120 is paid for £100 stock at 5 per cent. *Ans.* £4 3s. $2\frac{3}{4}$d.

13. If £500 stock, at $3\frac{1}{2}$ per cent., cost £454 2s. 6d. including brokerage, what will be the rate per cent. interest?

Ans. £3 17s. $0\frac{148}{173}$d.

14. £100 stock, at 4 per cent., can be bought for £98; and £100 stock, at 5 per cent., for £120; which is the better investment?

Ans. The interest on the former is $4\frac{4}{49}$ per cent., and on the latter $4\frac{1}{6}$.

EQUATION OF PAYMENTS.

58. The object of this rule is to determine the time at which several debts, payable at different times, may be discharged at once.

Common Rule.—Multiply each debt by the time at which it is due, then divide the sum of the products by the sum of the debts, and the quotient will be the time required.

1. If a debt of £300 be payable at 4 months, and another debt of £800 at 8 months; at what time may the whole be paid?

$$\text{The time} = \frac{300 \times 4 + 800 \times 8}{300 + 800} = \frac{7600}{1100} = 6\frac{10}{11} \text{ mon.} = 6 \text{ mon. } 27 \text{ d.}$$

2. If one person owe to another £360 payable in 6 months, and £540 payable in 16 months; in what time may the whole be paid, without loss to either party? *Ans.* 12 mon.

3. A merchant bought goods to the value of £900; £120 was to be paid at the end of 2 months; £400 at 4 months; and the remainder at 12 months; at what time may the whole be paid?

Ans. $7\frac{1}{9}$ mon., or 7 mon. 3 days.

4. Bought goods to the value of £800, of which £240 was to be paid at once, and the remainder at the end of 18 months; at what time may the whole be paid?

$$\text{In this case, the time} = \frac{240 \times 0 + 560 \times 18}{800} = 12\frac{3}{5} \text{ mon.}$$

5. Required the same as in the last example, when £460 is to be paid at once, and £340 at 8 months. *Ans.* 3 mon. 12 days.

6. A owes B £1200; of which £220 is due at 4 months; £340 at 12 months; and £640 at 18 months; at what time may the whole be paid? *Ans.* 13 mon. 22 days.

7. Required the same as in the last example, when £200 is due at 30 days; £250 at 60 days; and £120 at 85 days. *Ans.* 54 days.

The foregoing rule assumes that the interest of a debt, payable at a certain time hence, is the proper discount on that debt. In Ex. 1, let 5 be the rate of interest; then the Bankers' discount of the two debts will be $\frac{300 \times 4 \times 5}{12 \times 100} + \frac{800 \times 8 \times 5}{12 \times 100}$; and the discount on the whole payable at the time required in months will be $\frac{1100 \times \text{time} \times 5}{12 \times 100}$; but the discount in each case must be the same,

$$\therefore \quad \frac{1100 \times \text{time} \times 5}{12 \times 100} = (300 \times 4 + 800 \times 8) \times \frac{5}{12 \times 100}$$

$$\therefore \quad \text{Time} = \frac{300 \times 4 + 800 \times 8}{1100}$$

When the times of payment are short, the errors arising from the use of this simple rule are small. The following rule gives the *true* time of payment.

Rule.—Find the present value of the several debts at the given rate of interest (Art. 54); then find the time at which this sum will amount to the sum of the debts (Art. 51); this result will be the time required.

For example, let 6 be the rate of interest in Ex. 1, then

$$\text{Present value debts} = \frac{300 \times 100}{102} + \frac{800 \times 100}{104} = £1063{\cdot}348.$$

Now we have to find in what time this sum will amount to the whole debt.

Int. of £1063·348 for 1 year = £63·8; hence we have by Rule Art. 51,

$$\text{Time} = \frac{1100 - 1063{\cdot}348}{63{\cdot}8}\text{y.} = {\cdot}5744\text{y.} = 6 \text{ mon. } 26 \text{ days.}$$

8. Required the *true* time at which the following debts may be paid at once; £505 due at 3 months; £255 at 6 months; and £412 at 9 months; the rate of interest being 4 per cent.

Here we find the present value of the debts to be £1150, and the interest of this sum for 1 year to be £46; then as the sum of the debts is £1172, we find, the time required $= \frac{1172 - 1150}{46}\text{y.} = \frac{22}{46}\text{y.} = 5$ months 22 days.

9. Find the *true* times of payment in examples 2, 3, the rate of interest being 6 per cent. *Ans.* 11 mon. 26d.; 7 mon.

PROFIT AND LOSS.

59. By this rule we find how much is gained or lost by the buying and selling of goods.

1. Bought a cwt. of rice for £1 4s., and sold it at $3\frac{1}{4}$d. per lb.; required the gain on the whole.

	£	s.	d.
Selling price = 112 × $3\frac{1}{4}$d. =	1	10	4
Cost price =	1	4	0
Gain =		6	4

2. Bought $25\frac{1}{2}$ yards of linen for £1 14s. $1\frac{1}{2}$d., what is gained by selling it at 1s. $10\frac{3}{4}$d. per yard? *Ans.* 14s. $2\frac{1}{4}$d.

3. Bought 118 lbs. of tea, at 3s. 10d. per lb., and sold it at 3s. 5d., what is the loss on the whole? *Ans.* £2 9s. 2d.

4. Bought 25 articles at 11s. 6d., and sold them at 13s. 2d. each, what is the gain? *Ans.* £2 1s. 8d.

5. Bought 68 yards of flannel at 3s. 8d. per yard, at what price per yard must it be sold to gain £6?

Gain per yard = £6 ÷ 68 = 1s. 9d.

∴ Selling price per yard = 3s. 8d. + 1s. 9d. = 5s. 5d.

6. Bought 60 articles at 3s. 3d. each, at what price must they be sold to gain 15s.? *Ans.* 3s. 6d. each.

7. Sold 156 articles at 7s. 2d. each, and gained £23; required the cost price of each? *Ans.* 4s. $2\frac{1}{2}$d.

Given the prime cost and selling price, to find the gain or loss p. c.

Obs. Per-centages are always calculated on the prime cost.

8. Bought tea at 3s. 3d. per lb., and sold it at 3s. 8d. per lb., what is the gain per cent.?

Gain on 3s. 3d. or 39d. = 5d.

$\therefore$ Gain on £39 = £5

$$\therefore \quad ,, \quad £100 = £\frac{5 \times 100}{39} = £12\tfrac{32}{39}$$

$$= £12 \text{ 16s. } 4\tfrac{12}{13}\text{d.}$$

9. Bought goods at 1s. 9d. per lb., and sold them at 2s. 3d. per lb., what is the gain per cent.? *Ans.* $28\frac{4}{7}$.

10. Bought goods for £8, and sold them for £10, required the gain per cent.? *Ans.* 25.

11. Bought goods for £50, and sold them for £60, required the gain per cent.? *Ans.* 20.

12. If sugar be bought at £2 6s. 8d. per cwt., and sold at £2 11s. 4d., what is the gain per cent.? *Ans.* 10.

13. If tea be bought at 6s. 2d. per lb., and sold at 5s. 10d. per lb., what is the loss per cent.?

Here the loss on 74d. is 4d., therefore

Loss on £74 = £4

$$\therefore \quad ,, \quad £100 = £\frac{4 \times 100}{74} = £5\tfrac{15}{37} \text{ or } £5 \text{ 8s. } 1\tfrac{11}{37}\text{d.}$$

14. Bought goods at £24 3s. 9d. per ton, and sold them at £22 15s. per ton, what is the loss per cent., and what is the loss on the sale of 19 tons $18\frac{1}{2}$ cwt.? *Ans.* £5 18s. $10\frac{1}{4}$d.; £28 12s. 10d.

Given the prime cost and gain or loss per cent., to find the selling price.

15. Bought goods at 8s. 9d. per lb., how must they be sold to gain 16 per cent.?

Here the selling price of £100 is £116, therefore

Selling price of 100d. = 116d.

$$\therefore \text{ Selling price of 8s. 9d. or 105d.} = \frac{116\text{d.} \times 105}{100} = 10\text{s. } 1\tfrac{3}{4}\text{d.}$$

16. Bought goods for £6, how must they be sold to gain 5 per cent.? *Ans.* £6 6s.

17. Bought goods for £15, how must they be sold to lose 5 per cent.?

Selling price of £100 = £95

$$\therefore \quad ,, \quad £15 = £\frac{95 \times 15}{100} = £14 \text{ 5s.}$$

18. Bought goods for £1 1s. 3d., at what price must they be sold so that the loss may be 7 per cent.? *Ans.* 19s. 9d.

19. If sugar cost £2 10s. per cwt., how must it be sold to gain 5 per cent., and how to gain $7\frac{1}{2}$ and 10 per cent.?

Ans. £2 12s. 6d.; £2 13s. 9d.; £2 14s.

Given the gain or loss per cent. and the selling price to find the cost price, &c.

20. If 13 per cent. be gained by selling tea at 3s. 8d. per lb., what was the cost price?

Here the cost price of £113 is £100, therefore

Cost price of 113d. = 100d.

$$\therefore \text{ Cost price of 3s. 8d. or 44d.} = \frac{100 \times 44}{113}\text{d.} = \text{3s. } 2\tfrac{2}{5}\text{d.}$$

21. A person sold goods for £169, and gained 30 per cent., what was the cost price? *Ans.* £130.

22. Sold goods for £144, and lost 10 per cent., what was the cost price? *Ans.* £160.

23. Sold cloth at 2s. 5d. per yard, and gained $20\frac{5}{6}$ per cent., what was the cost price? *Ans.* 2s.

24. If 10 per cent. be gained by selling goods for £33, at what price must they be sold to gain 25 per cent.?

As in the foregoing examples, we find

$$\text{Cost price} = £\frac{100 \times 33}{110} = £30$$

$$\text{Selling price } £100 = £125$$

$$\therefore \quad ,, \quad £30 = £\frac{125 \times 30}{100} = £37 \text{ 10s. } \textit{Ans.}$$

Or thus. Omitting the last step in the foregoing process, we have,

Selling price $£100 \times \frac{33}{110} = £125 \times \frac{33}{110} = £37$ 10s.

By this method we avoid the calculation for the value of the cost price.

25. If 4 per cent. be gained by selling tea at 4s. 4d. per lb., at what price must it be sold to gain 10 per cent.? *Ans.* 4s. 7d.

26. If 7 per cent. be gained by selling articles at 5s. 6d. each, how must they be sold to gain 25 per cent.? *Ans.* 6s. 5d.

27. If 25 per cent. be gained by selling sugar at £2 10s. per cwt., how much is gained per cent. by selling it at £2 2s.?

$$\text{Here,} \quad \text{Cost price} = £\frac{100 \times 2\frac{1}{2}}{125} = £2$$

$$\therefore \text{ Gain on } £2 = 2\text{s.}$$

$$\therefore \quad ,, \quad £100 = \frac{2\text{s.} \times 100}{2} = £5. \ \textit{Ans.}$$

28. If 5 per cent. be gained by selling articles at £1 11s. 6d. each, how much is gained per cent. by selling them at £1 12s. 3d. each?

Ans. $7\frac{1}{2}$.

Given the cost and selling prices to find the number of articles.

29. Bought articles at 3s. 3d. each, and sold them at 3s. 6d.; how many of them must be sold to gain £5?

Gain on 1 article = 3s. 6d. — 3s. 3d. = 3d.;

$\therefore$ No. articles = £5 ÷ 3d. = $5 \times 20 \times 12 \div 3 = 400$.

30. How many of them must be sold to gain 12s. 6d.? *Ans.* 50.

31. Bought a package of tea for £27, and sold it at 3s. 9d. per lb., and thereby gained 25 per cent.; how many lbs. were there?

Cost price of 125d. = 100d.; ∴ Cost price of 1d. = $\frac{4}{5}$d.
∴ Cost price of 3s. 9d. or 45d. = $\frac{4}{5}$d. × 45 = 3s.;
∴ No. lbs. = £27 ÷ 3s. = 180.

32. Find the no. lbs., supposing the gain to be 20 p. c. *Ans.* 172$\frac{4}{5}$.

33. Bought coffee at 18d. per lb., and the same quantity of another sort at 12d. per lb., and sold the mixture at 16$\frac{1}{2}$d. per lb., and thereby gained 8s. 4d.; how many lbs. of each sort were there?

Cost price mixture per lb. = $\frac{1}{2}$ (18d. + 12d.) = 15d.;
∴ Gain per lb. = 16$\frac{1}{2}$d. — 15d. = 1$\frac{1}{2}$d.;
∴ Total no. of lbs. = 8s. 4d. ÷ 1$\frac{1}{2}$d. = 62$\frac{2}{3}$;
and no. lbs. of each sort = $\frac{1}{2}$ of 62$\frac{2}{3}$ = 31$\frac{1}{3}$.

34. Find the no. lbs., supposing the cost of the two sorts of coffee to be at 18$\frac{1}{2}$d. and 13d. per lb. *Ans.* 66$\frac{2}{3}$.

35. A woman bought some apples at 2 for 1d., and as many at 4 for 1d., and sold them at 5 for 2d., and found that she had gained 1s.; how many of each sort did she buy?

Cost price of each apple = $\frac{1}{2}$ ($\frac{1}{2}$d. + $\frac{1}{4}$d.) = $\frac{3}{8}$d.;
Selling price of each apple = $\frac{2}{5}$d.;
∴ Gain on each apple = $\frac{2}{5}$ — $\frac{3}{8}$ = $\frac{1}{40}$d.;
but she gained 12d. in selling the total number,
∴ Total number = 12d. ÷ $\frac{1}{40}$d. = 12 × 40 = 480;
and the no. apples of each sort = $\frac{1}{2}$ of 480 = 240.

36. Find the number, supposing she sold them at 7 for 3d. *Ans.* 112.

60. PARTNERSHIP OR FELLOWSHIP, AND DIVISION IN RATIOS.

1. Two partners, A and B, gain by trade £35; the capital put in by A was £250, and that by B £170; what is their respective shares of the profit?

Here, the total sum in trade is £420, with which £35 are gained; then

Gain on £420 = £35
∴ „ £1 = £$\frac{35}{420}$
∴ „ £250 = £$\frac{35 \times 250}{420}$ = £20 16s. 8d. A's share.
∴ „ £170 = £$\frac{35 \times 170}{420}$ = £14 3s. 4d. B's share.

2. A's stock or capital £875, B's £625; whole gain £282 16s.
Ans. A's £164 19s. 4d.; B's £117 16s. 8d.

3. Three merchants, A, B, and C, make a joint stock, of which A contributes £350, B £500, C £800. What is the share of each in a gain of £440? *Ans.* A's £93 6s. 8d.; B's £133 6s. 8d.; C's £213 6s. 8d.

4. A bankrupt owes to A £413 6s.; to B £631 14s. 9d.; to C £362 7s. 11d.; to D £500; and to E £121 8s. 2d.; but his whole estate

amounts only to £1050. How much can he pay in the pound, and what is the divend of each creditor?

Here the sum of the debts is £2028 16s. 10d. = 486922d., and £1050 = 252000d., that is, the sum to pay 486922d. is 252000d., or what would be the same thing

Sum to pay £486922 = £252000

$\therefore$,, $£1 = £\frac{252000}{486922} = £\cdot5175367$ or 10s. 4d.

that is, the sum paid in the pound will be £·5175367 or 10s. 4d.

$\therefore$ A's dividend = £413 6s. $\times$ ·5175367 = £213 17s. 11½d.

This is best calculated by aliquot parts as shown in the margin, where we multiply by 413, and take parts for the 6s.

	£
	·5175367
	413
	213·7426571
5s. = ¼	·1293841
1s. = ⅕	·0258768
	213·8979180
	= £213 17s. 11½d.

In like manner, B's dividend = £631 14s. 9d. $\times$ ·5175367 = £326 18s. 11½d.; and proceeding in the same manner, we find, C's £187 11s. 0¾d.; D's £258 15s. 4¼d.; and E's £62 16s. 8d.

The sum of all these dividends is £1050 (less by a farthing), which proves the accuracy of the calculation.

By the method of Ex. 1, we find

$$\text{A's dividend} = \frac{£413\ 6s. \times 252000}{486922} = £413\ 6s. \times \cdot5175367 \text{ as before;}$$

and so on to the other dividends.

The following questions may be solved in a similar manner.

5. A's stock £297 16s. 4d., B's £350 15s. 9d.; whole gain £294 0s. 10½d. *Ans.* A's £135 0s. 3½d.; B's £159 0s. 7d.

6. A's stock £174 8s. 3d., B's £402 5s. 8d., C's £310 16s. 1d.; whole gain £397 19s. 4d.

Ans. A's £78 4s. 2d.; B's £180 7s. 9¼d.; C's £139 7s. 4½d.

7. A bankrupt owes to A £450; to B £430; to C £320; to D £75; and to E £50; but his whole estate amounts only to £625. What is the dividend of each creditor?

Ans. A's £212 5s. 3¼d.; B's £202 16s. 7¼d.; C's £150 18s. 10¼d.; D's £35 7s. 6½.; E's £23 11s. 8¼d.

8. Divide 72 into two parts, which shall be as 5, 3. *Ans.* 45, 27.

Here 5 + 3 = 8. Now supposing 72 to be divided into 8 equal parts, we have, 1 part = ⅛ of 72 = 9, $\therefore$ 5 parts = 9 $\times$ 5 = 45, and 3 parts = 9 $\times$ 3 = 27.

9. Divide 84 into 3 parts, which shall be as 3, 4, 5. *Ans.* 21, 28, 35.

10. Divide £18 into shares, so that A may have 3 shares, and B 5 shares. *Ans.* £6 15s.; £11 5s.

11. How much copper and tin are contained in a bell weighing 300 lbs., supposing the bell-metal to be composed of 3 parts of copper and 1 part of tin?

Dividing the weight of metal into 4 equal parts, we have

Wt. 1 part $= \frac{1}{4}$ of 300 lbs. $=$ 75 lbs. wt. of tin
,, 3 parts $=$ 75 lbs. $\times$ 3 $=$ 225 lbs. wt. of copper.

12. Gunpowder is composed of 76 parts of nitre, 14 of charcoal, and 10 of sulphur: how many pounds of each ingredient will be requisite to make a cwt. of powder? *Ans.* $85\frac{3}{25}$; $15\frac{17}{25}$; $11\frac{1}{5}$.

13. A crew of a vessel consisted of a captain, 2 officers, and 7 sailors; it is required to divide a prize of £289 amongst them according to each man's pay, viz., at the rate of 10s. for the captain, 5s. for each officer, and 2s. for each sailor.

Here, 10s. + 5s. $\times$ 2 + 2s. $\times$ 7 $=$ 34s.; then

Prize money on 34s. $=$ £289

$\therefore$,, 10s. $= £\frac{289 \times 10}{34} =$ £85, captain's

,, 5s. $=$ £42 10s., each officer's

,, 2s. $=$ £17, each sailor's.

Or thus. Sailor's share in 34s. $=$ 2s.

$\therefore$,, £1 $= £\frac{2}{34}$

$\therefore$,, £289 $= £\frac{2 \times 289}{34} =$ £17.

And so on to the others.

14. Divide £4 5s. amongst 3 bricklayers and 2 labourers according to their rate of wages, a bricklayer's wages being 4s. per day and a labourer's 2s. 6d.

Ans. £1 to each bricklayer; 12s. 6d. to each labourer.

15. Divide 16s. 4d. between A and B, so that A's share may be $\frac{2}{5}$ of B's.

Supposing B to have 5 shares; A will have 2 shares, because $\frac{2}{5}$ of 5 is 2; then dividing the sum into 7 shares, we have

Value 7 shares $=$ 16s. 4d.

$\therefore$,, 5 ,, $=$ 16s. 4d. $\times \frac{5}{7} =$ 11s. 8d. B's

$\therefore$,, 2 ,, $=$ 16s. 4d. $\times \frac{2}{7} =$ 4s. 8d. A's.

16. Divide £69 amongst A, B, and C, so that A's share may be $\frac{2}{3}$ of B's, and C's $\frac{1}{4}$ of B's. *Ans.* A's £23; B's £36; C's £9.

17. Divide £66 10s. amongst 3 men, 2 women, and 6 boys, so that a woman may have twice as much as a boy, and a man as much as a boy and a woman together.

Taking each boy's portion as *one share*, each woman will have 2 shares, and each man 3 shares; and therefore the number of shares $= 1 \times 6 + 2 \times 2 + 3 \times 3 = 19$; then

Value 19 shares $=$ £66 10s.

$\therefore$,, 1 ,, $= \frac{£66\ 10s.}{19} =$ £3 10s., boy's share;

$\therefore$,, 2 ,, $=$ £7, woman's share;

$\therefore$,, 3 ,, $=$ £10 10s., man's share.

18. Divide £20 amongst A, B, C, so that A shall have twice as much as B, and B thrice as much as C. *Ans.* £12; £6; £2.

19. Divide £27 amongst A, B, and C, so that A shall have one-half as much as B, and C shall have twice as much as A and B together.

Ans. £3; £6; £18.

20. A road passes through two parishes; in the first parish there are 4 miles of it; and in the second 5 miles; if the repairs of the road cost £63, what must each parish contribute?

Here,	Cost on 9 miles			= £63
∴	„	4	„	= £7 × 4 = £28
∴	„	5	„	= £7 × 5 = £35

21. A road passes through three parishes; in the first parish there are 3 miles of it; in the second 2 miles; and in the third $5\frac{1}{2}$ miles; if the road cost £42 in repairs, what must each parish contribute? *Ans.* £12; £8; £22.

22. Suppose a traveller to go from London to York at the rate of 5 miles an hour, and another at the same time from York to London at the rate of 3 miles an hour; where will they meet, the distance between the places being 200 miles? *Ans.* 75 miles from York.

Here we have to divide 200 into two parts which are as 5, 3.

23. Divide a proverty, valued £240, amongst A, B, C, in the following manner: A to have $\frac{2}{3}$ of the whole, and the remainder to be divided between B and C, so that B shall £10 more than C. *Ans.* A's £160; B's £45; C's £35.

24. Divide unity into three parts, which shall be as 2, 4, 5. *Ans.* $\frac{2}{11}$, $\frac{4}{11}$, $\frac{5}{11}$.

25. A owns ·45 of a ship and B the rest; the difference of the value of their shares is £90·7; what is the value of the ship?

B's share = 1 — ·45 = ·55; ∴ Diff. of their shares = ·55 — ·45 = ·1;
∴ Value $\frac{1}{10}$ = £90·7; ∴ Value whole = £907.

26. What is the value of the ship if A owns ·43? *Ans.* £$647\frac{6}{7}$.

27. A mass of copper and tin, weighing 64 lbs., contains 5 lbs. of copper for every 3 lbs. of tin; how much copper must be added to the mass, so that there may be 9 lbs. of copper for every 4 lbs. of tin?

We shall first find the respective weights of the copper and tin in the mass. Dividing the weight of the mass into 8 equal parts, 5 of them will be copper and 3 of them tin, that is,

Weight of copper = $\frac{5}{8}$ of 64 lbs. = 40 lbs., and
„ tin = $\frac{3}{8}$ of 64 lbs. = 24 lbs;

but, in the new compound, the weight of the copper is $\frac{9}{4}$ of the weight of tin,

∴ Weight of copper = $\frac{9}{4}$ of 24 lbs. = 54 lbs.;
∴ Weight of copper to be added = 54 lbs. — 40 lbs. = 14 lbs.

28. Required the weight of copper that must be added, when the weight of the mass is 48 lbs. *Ans.* $10\frac{1}{2}$ lbs.

29. In 100 parts of air there are 21 of oxygen, and 79 of nitrogen; how many feet of oxygen are there in an apartment containing 3000 feet of air? *Ans.* 630.

30. Air expands the 490th part of its volume, at 32°, for every degree that it is increased in temperature. If 70 feet of air has a temperature of 32°, what would be its volume when its temperature is 46°?

Here the increase of temperature = 46° — 32° = 14°; then

Expansion for 1° = $\frac{70}{490}$ = $\frac{1}{7}$; ∴ Expansion for 14° = $\frac{1}{7}$ × 14 = 2;
∴ The volume = 70 + 2 = 72 ft.

31. 100 c. in. of air is at the temperature of 42°; what would be its volume at it 32°?

Here the decrease of temperature = 42° — 32° = 10°. Now if 1 be the volume at 32°; then the volume at 42° = $1 + \frac{1}{490} \times 10 = 1\frac{1}{49}$, that is, the volume at 42° will be $1\frac{1}{49}$ times the volume at 32°;

$$\therefore\ 1\tfrac{1}{49} \text{ times the volume at } 32° = 100 \text{ c. in.,}$$
$$\therefore\ \text{the volume at } 32° = 100 \div 1\tfrac{1}{49} = 98 \text{ c. in.}$$

32. What would be the volume of this air at 62°?

Here the increase of temperature = 62° — 32° = 30°;

$$\therefore\ \text{Vol. of the air at } 62° = 98 + \tfrac{98}{490} \times 30 = 104 \text{ c. in.}$$

33. 162 c. in. of air is at the temperature of 82°, what would be its volume at 62°? *Ans.* Vol. at 32° = 147, vol. at 62° = 156.

34. The pressure or elasticity of air is inversely as the space which it occupies. Air occupies the space of 72 c. in. when the pressure upon it is measured by a mercury column of 30 inches; what will be its volume when the pressure upon it is 27 inches?

Vol. air at 30 in. pressure = 72 c. in.
„ 1 „ = 72 × 30 = 2160
„ 27 „ = $\frac{1}{27}$ of 2160 = 80 c. in.

35. Air occupies the space of 124 c. in. when the pressure is 28 in.; what will be its volume when the pressure is 31 in.? *Ans.* 112 c. in.

36. Air occupies the space of 118 c. in. when the pressure upon it is measured by a mercury column of 30·5 inches and temperature 52°; what will be its volume when the pressure is 29·5 inches and the temperature 32°?

Here we shall first find the change of volume due to the change of pressure. Proceeding as in Ex. 34, we find

$$\text{Volume at 29·5 pressure} = \frac{118 \times 30·5}{29·5} = 122 \text{ c. in.}$$

Now we have to reduce 122 c. in. of air to the volume corresponding to 32°. The decrease of temperature = 52° — 32° = 20°; then, proceeding as in Ex. 31, if 1 be the vol. at 32°, the vol. at 52° = $1 + \frac{1}{490} \times 20 = 1\frac{2}{49}$, that is,

$$1\tfrac{2}{49} \text{ times the vol. at } 32° = 122 \text{ c. in.}$$
$$\therefore\ \text{the vol at } 32° = 122 \div 1\tfrac{2}{49} = 117\tfrac{11}{51} \text{ c. in.}$$

37. What would be the volume of this air at 83°?

Here the increase of temp. = 83° — 32° = 51°; then

$$\text{Vol. at } 83° = 117\tfrac{11}{51} + 117\tfrac{11}{51} \times \tfrac{51}{490} = 129\tfrac{106}{255} \text{ c. in.}$$

38. Air occupies the space of 159 c. in., at 72° temperature, under a pressure of 28 inches mercury column; what will be its volume when the temperature is 81°, and the pressure upon it is 30 inches?

Ans. Vol. at 32°, pressure 30 = 137·2; Vol. at 81° = 150·92.

61. *Rule of Compound Fellowship.*—When the times, during which the different stocks remain in trade, are unequal, we multiply each stock by the time of its continuance, and then proceed as before.

1. A's stock of £240 was 4 months in trade; B's stock of £120 was 10 months; required the share of each in a gain of £60.

The value of any given stock (as regards the interest of the trading firm) is proportional to the time of its continuance; thus the use of £100 stock for 4 months will be the same as the use of £400 for 1 mon.

$$\begin{aligned} & \quad\ \text{£} \\ \text{£}240 \times 4 &= \ \ 960 \text{ A's stock for 1 mon.} \\ \text{£}120 \times 10 &= 1200 \text{ B's} \quad ,, \quad ,, \\ & \overline{2160} \text{ whole} \quad ,, \quad ,, \end{aligned}$$

Then proceeding as in the case of single fellowship, we find

$$\text{A's share} = \text{£}\frac{60 \times 960}{2160} = \text{£}26\ 13s.\ 4d.$$

$$\text{B's share} = \text{£}\frac{60 \times 1200}{2160} = \text{£}33\ 6s.\ 8d.$$

2. A's stock of £170 was 4 months in trade; B's stock of £280 was 3 months; required the share of each in a gain of £125.

Ans. A's £55 18s. 5d.; B's £69 1s. 6¾d.

3. A's stock of £124 6s. 3d. was 10 months in trade; B's stock of £335 was 3 months; C's stock of £256 3s. 9d. was 6 months; required the share of each in a gain of £219 19s. 4d.

Ans. A's £72 4s. 9½d.; B's £58 8s. 0½d.; C's £89 6s. 5¾d.

4. Three persons rent a grass field at £54; A puts in 20 oxen for 5 months; B 25 for 8 months; and C 15 for 4 months; what part of the rent must each pay? *Ans.* A's £15; B's £30; C's £9.

5. A's stock of £450 was 8 months in trade; B's stock of £350 was 12 months; now A's share of the gain was £72; required B's share.

Here we find, Gain on £3600 = £72

$$\therefore \quad ,, \quad \text{£}4200 = \text{£}\frac{72 \times 4200}{3600} = \text{£}84 \text{ B's.}$$

6. A's stock of £60 was 3 months in trade; B's stock of £50 was 4 months; now A's share of the gain was £36; required the whole gain. *Ans.* £76.

7. Two graziers rent a grass field at £18 10s.; A put in 60 sheep for 10 months, and 8 oxen for 5 months; B put in 20 sheep for 6 months, and 20 oxen for 9 months; now it was agreed between them that 5 sheep could eat as much as 1 ox; what part of the rent ought each to pay? *Ans.* A's £8 2s. $7\frac{59}{61}$d.; B's £10 7s. $4\frac{2}{61}$d.

BARTER.

62. Barter is the exchange of one kind of goods for another.

1. How much tea at 4s. 6d. per lb. must be given for 63 yards of cloth at 4s. per yard?

Value cloth = 4s. × 63 = 252s.; Value tea per lb. = 4½s.

$$\therefore \quad \text{No. lbs. tea} = \frac{252}{4\frac{1}{2}} = 56.$$

2. How much tea at 2s. 3d. per lb. must be given for 45 yds. of linen at 1s. 6d. per yd.? *Ans.* 30 lbs.

3. How much rum at 9s. 2d. per gallon must be given for 20 bales of cloth at £75 5s. per bale? *Ans.* $3283\frac{7}{11}$ gals.

4. 75 yds. of cloth were given in exchange for 175 lbs. of tea, at 6s. 6d. per lb.; required the value of the cloth per yard.

$$\text{Value tea} = 6s.\ 6d. \times 175;\ \text{No. yds. cloth} = 75;$$

$$\therefore\ \text{Value cloth per yd.} = \frac{6s.\ 6d. \times 175}{75} = 15s.\ 2d.$$

5. $673\frac{1}{4}$ yds. of linen were given in barter for $193\frac{1}{2}$ yds. of cloth at 6s. 3d. per yd.; required the value of the linen per yd. *Ans.* 1s. $9\frac{1}{2}$d.

6. Exchanged 84 qrs. of wheat, at 56s. per qr., and received $\frac{5}{8}$ of the value in sugar at $4\frac{1}{4}$ per lb., and the rest in rum at 16s. 8d. per gallon; how much of each was given? *Ans.* $105\frac{21}{25}$ gal.; $8301\frac{3}{17}$ lbs.

7. Exchanged $14\frac{1}{2}$ yds. of cloth, at 15s. 4d. per yd., and got in return equal quantities of coffee and chocolate, the coffee being at 10d. per lb., and the chocolate at 1s. 7d.; how much was received of each? *Ans.* 92 lbs. of each.

8. Bought 6 qrs. of wheat at £2 16s. per qr., for which I paid in cash £6 16s., and the remainder in tea at 2s. 6d. per lb.; how many lbs. of the tea had I to give? *Ans.* 80.

9. Bought 20 cwts. of sugar at £2 10s. per cwt., for which I paid in cash £17, and the remainder in cloth at 7s. 6d. per yd.; how many yards had I to give? *Ans.* 88.

EXCHANGE.

63. The object of this rule is to find how much money of one country is equal to a given sum of another. The *par of exchange* is the fixed or intrinsic value of the one compared with that of the other; but the *rate* or *course of exchange* is continually varying according to the circumstances of trade, &c. Some places have two kinds of money one called *banco*, or bank money, the other currency or current money. Bills of exchange are tranacted in the former, and accounts are usually kept in the latter. The bank money is of purer metal than the currency, and hence it bears a premium of 3 to 5 per cent; this premium is called agio. The American pound, being in paper money, is of much smaller value than the pound British. Extended tables are given of the money, weights, and measures of different countries in works expressly written on the subject; the following tables of money will be found sufficient for the purpose of this work.

United States.—1 dollar = 10 dimes = 100 cents = 1000 mills.

France.—1 napoleon = 20 francs; 1 franc = 10 decimes = 100 centimes.

Amsterdam.—8 pennings = 1 grote or penny flemish; 2 grotes or 16 pennings = 1 stiver; 20 stivers = 1 florin or guilder; 6 florins = 1 pound. Also 12 grotes or pence = 1 skilling; 20 skillings = 1 pound; 50 stivers = 1 rix dollar.

Portugal.—1 milree = 1000 rees, or $2\frac{1}{2}$ crusadoes; 1 moidore = 4800 rees.

The following examples will sufficiently illustrate the principles on which calculations of this kind are conducted.

1. Reduce £492 3s. 4d. to American dollars, exchange at 4s. 7d. per dollar.

$$\text{No. dollars} = \frac{£492\ 3s.\ 4d.}{4s.\ 7d.} = 2147{\cdot}636 = 2147 \text{ dol. } 63{\cdot}6 \text{ cents.}$$

2. Reduce £246 1s. 8d. to American dollars, exchange at 4s. 7d. per dollar. *Ans.* 1073 dol. 82 cents.

3. Reduce £468 2s. 8d. to American dollars, exchange at 4s. $6\frac{1}{2}$d. per dollar. *Ans.* 2061 dol. 5·5 cents.

4. Reduce 1373 dol. 15 c. to Eng. money, exchange at 4s. $3\frac{1}{2}$d. per dollar.

Here the value in Eng. money obviously = 4s. $3\frac{1}{2}$d. × 1373·15. The calculation in this case may be most conveniently performed by the method of aliquot parts, as shown in the margin.

	1373·15
	4s.
	5492·60
3d. = $\frac{1}{4}$	343·2875
$\frac{1}{2}$d. = $\frac{1}{6}$	57·2145
	5893·1020s.
	= £294 13s. 1d.

5. Reduce 2805 dol. 71 cents, to Eng. money, exchange at 4s. $5\frac{1}{2}$d. per dollar. *Ans.* £625 8s. $9\frac{1}{2}$d.

6. Reduce 3796 dol. 85 cents, to Eng. money, exchange at 4s. 7d. per dollar. *Ans.* £870 2s. $2\frac{3}{4}$d.

7. Reduce £191 4s. Eng. to Amer. currency, at 71 per cent. premium.

Eng. £100 = £171 Amer. currency

∴ „ £1 = £$\frac{171}{100}$ „ „

∴ „ £$191\frac{1}{5}$ = £$191\frac{1}{5}$ × $\frac{171}{100}$ = £326 19s. $0\frac{1}{2}$d.

8. Reduce £183 12s. Eng. to Amer. currency, at 52 per cent.

Ans. £279 1s. $5\frac{7}{25}$d.

9. Reduce £398 6s. 9d. Amer. currency to Eng. money, exchange at 69 per cent. premium on the British money.

Here £100 sterling, or Eng. money, is equivalent to £169 American currency, hence we write,

£169 Amer. currency = £100 sterling

∴ £398 6s. 9d. „ = £398 6s. 9d. × $\frac{100}{169}$ = £235 14s. $0\frac{1}{2}$d.

10. Reduce £1693 1s. 6d. Amer. currency to Eng. money, exchange at 66 per cent. premium on the British money. *Ans.* £1019 18s. 6d.

11. Reduce £182 3s. 9d. to francs, &c., exchange at 23 fr. 50 c. per £.

23 fr. 50 c. = 23·5 fr.; £182 3s. 9d. = 43725d.;

Value £1 or 240d. = 23·5 fr.

$$\therefore \text{ Value } 43725\text{d.} = \frac{23{\cdot}5 \times 43725}{240} \text{ fr.} = 4281 \text{ fr. } 40 \text{ c. } \textit{Ans.}$$

Or thus by aliquot parts.

Value £1 =	23·5 fr.
	182
Value £182 =	4277·0
Value 3s. 4d. or $\frac{1}{6}$ of £ =	3·9166
Value 5d. or $\frac{1}{8}$ of 3s. 4d. =	·4895
	4281·4062 = 4281 fr. 40 c.

12. Reduce £98 8s. 7d. to francs, &c., exchange at 23 fr. 57 c. per £.
Ans. 2154 fr. 98·54 c.

13. Reduce £434 8s. 9d. to francs, &c., exchange at 23 fr. 60 c. per £.
Ans. 10252 fr. 72½ c.

14. Reduce 1433 fr. 74 c. to Eng. money, exchange at 24 fr. 36 c. per £.

$$\text{Here } 1433{\cdot}74 \text{ fr.} = £\frac{1433{\cdot}74}{24{\cdot}36} = £58\ 17\text{s. } 1\tfrac{1}{2}\text{d.}$$

15. Reduce 37823 fr. 59 c. to Eng. money, exchange at 24 fr. 51 c. per £. *Ans.* £1543 3s. 9½d.

16. Reduce £320 to francs, exchange at 25 fr. 25 c. per £.
Ans. 8080.

17. Reduce £241 18s. 5½d. to francs, exchange at 24 fr. 37 c. per £.
Ans. 5895 fr. 67 c.

18. Reduce 368 fr. 15 c. to Eng. money, exchange at 25 fr. 35 c. per £. *Ans.* £14 10s. 5¼d.

19. A gentleman's bill amounts to 36 francs 95 cents; how much English money will be required to settle it, at 25 fr. 12 c. for £1?
Ans. £1 9s. 5d.

20. Reduce £193 8s. 8d. to Flemish money, exchange at 37 skillings 2 grotes Flemish per pound sterling.

Here £193 8s. 8d. = 46424d.; £1 = 240d.; 37 sk. 2 gr. = 3568 pennings; then

$$\text{Value } 240\text{d.} = 3568 \text{ pennings}$$

$$\therefore \quad ,, \quad 46424\text{d.} = \frac{3568 \times 46424}{240} \text{ pen.} = 690170 \text{ pen.}$$

$$= 2156 \text{ fl. } 15 \text{ st. } 10 \text{ pen., or } 359 \text{ pounds } 9 \text{ sk. } 3 \text{ gr. } 2 \text{ pen.}$$

Here the pennings are reduced to stivers by dividing by 16, and the stivers to florins by dividing by 20. Or the pennings are reduced to grotes by dividing by 8, the grotes to skillings by dividing by 12, and the skellings to pounds by dividing by 20.

21. Reduce 1237 florins 15 stivers 4 pennings to Eng. money, exchange at 36 skellings 10 grotes per pound sterling.

Here the given sum = 396034 pen.; 36 sk. 10 gr. = 3536 pen.; then

$$\text{Value } 3536 \text{ pen.} = £1$$

$$\therefore \quad ,, \quad 396034 \text{ pen.} = £\tfrac{396034}{3536} = £112\ 0\text{s. } 0\tfrac{30}{221}\text{d.}$$

22. Reduce £112 19s. 9¼d. to Flemish money, exchange at 34 sk. 6 gr. per £. *Ans.* 1169 fl. 8 st. 10 pen.

23. Reduce 361 pounds 9 sk. 2 gr. Flemish to Eng. money, exchange at 34 sk. 6 gr. per £. *Ans.* £209 10s. $9\frac{177}{207}$d.

24. Reduce 3135 fl. 5 st. 15 pen. Flemish to Eng. money, exchange at 34 sk. 11 gr. per £. *Ans.* £299 6s. 3d.

25. Reduce 1743 fl. 8 st. 1½ gr. Flemish to Eng. money, exchange at 35 sk. 3 gr. per £. *Ans.* £164 17s. 3¼d.

26. How much currency is equal to 399 florins 8½ stivers Flemish Banco, agio at 3½ per cent.?

Here, 399 fl. 8½ st. = 7988½ st.; and

$$100 \text{ Bank money} = 103\tfrac{1}{2} \text{ currency}$$

$$\therefore \quad 7988\tfrac{1}{2} \quad ,, \quad = \frac{103{\cdot}5 \times 7988{\cdot}5}{100} \text{ st.} = 413 \text{ fl. } 8 \text{ st. } 1\tfrac{1}{2} \text{ pen.}$$

27. How much currency is equal to 24375 florins Flemish banco, agio at $4\frac{3}{4}$ per cent.? *Ans.* 25532·8125 fl.

28. Reduce 219 florins 17 stivers Flemish currency to banco, agio at $3\frac{3}{4}$ per cent.

Here, 219 fl. 17 st. = 4397 st.; and

$$103\tfrac{3}{4} \text{ currency} = 100 \text{ banco}$$

$$\therefore\ 4397 \text{ st.}\quad ,, \quad = \frac{100 \times 4397}{103\frac{3}{4}} \text{ st.} = \frac{1758800}{415} \text{ st.}$$

$$= 4238 \text{ st. } 1 \text{ pen.} = 211 \text{ fl. } 18 \text{ st. } 1 \text{ pen.}$$

In dividing by 415 the remainder is multiplied by 16 to bring the stivers to pennings.

29. Reduce 1966 florins currency to florins banco, agio at $2\frac{1}{4}$ per cent. *Ans.* 1922·7383 fl.

30. Reduce £62 17s. 3d. to Flemish currency, exchange at 36 sk. 3 gr. per £ sterling, agio at 4 per cent.

£62 17s. 3d. = 15087d.; 36 sk. 3 gr. = $36\frac{1}{4}$ sk.; then

$$100 \text{ banco} = 104 \text{ currency}$$

$$\therefore\ 1\ ,, \quad = 1\cdot04\ ,,$$

$$\therefore\ 15087\text{d.}\ ,, \quad = 1\cdot04 \times 15087\text{d.}$$

$$\text{Value } 240\text{d.} = 36\tfrac{1}{4} \text{ sk.}$$

$$\therefore\ \text{Value } 1\cdot04 \times 15087\text{d} = \frac{36\frac{1}{4} \times 1\cdot04 \times 15087}{240} \text{ sk.} = 2369\cdot91625 \text{ sk.}$$

$$= 118 \text{ pounds } 9 \text{ sk. } 10\cdot995 \text{ gr.} \quad \textit{Ans.}$$

Or the result in florins = $2369\cdot91625 \times \frac{6}{20} = 710\cdot974875$ florins.

31. Reduce £810 sterling to Flemish currency, exchange at 35 sk. 6 gr., agio at 3 per cent. *Ans.* 8885·295 florins.

32. Reduce 1042 florins 8 stivers currency to Eng. money, agio at $4\frac{1}{4}$ per cent, exchange at 37 sk. 8 gr. Flemish per £ sterling.

1042 fl. 8 st. = 41696 gr.; 37 sk. 8 gr. = 452 gr.;

$$104\tfrac{1}{4} \text{ currency} = 100 \text{ bank money}$$

$$\therefore\ 41696 \text{ gr.}\quad ,, \quad = \frac{100 \times 41696}{104\frac{1}{4}} \text{ gr. bank money.}$$

But 452 gr. = £1;

$$\therefore\ \text{Eng. money} = £\frac{4169600}{104\frac{1}{4}} \div 452 = £88 \text{ 9s. } 8\tfrac{3}{4}\text{d.}$$

33. Reduce 1965 florins currency to Eng. money, agio at $2\frac{3}{4}$ per cent., exchange at 35 sk. 2 gr. per £ sterling. *Ans.* £181 5s. 5d.

34. Reduce £239 18s. 9d. to milrees Portuguese, exchange at 5s. $4\frac{1}{2}$d. per milree. Also £286 to milrees, at 5s. 8d. per milree. *Ans.* 892·7905; 1009·411.

35. Reduce 121·9685 milrees Portuguese to Eng. money, exchange at 5s. $3\frac{1}{2}$ per milree. *Ans.* £32 5s. 5d.

36. Reduce $2503\frac{3}{4}$ piastres Spanish to Eng. money, exchange at $40\frac{1}{2}$d. per piastre. *Ans.* £422 10s. $6\frac{3}{8}$d.

37. Reduce £190 to Russian rubles, exchange at 3s. 4d. per ruble. *Ans.* 1140 rubles.

38. Reduce 15708 rubles Russian to Eng. money, exchange at 3s. $4\frac{1}{2}$d. per ruble. *Ans.* £2650 14s. 6d.

ARBITRATION OF EXCHANGE.

64. Instead of conducting the exchange directly between two given places, merchants sometimes find it more advantageous to exchange with some intermediate place or places; by Arbitration of Exchange we find the value of any sum of money of the first place in that of the last through the medium of the other places.

1. Required the value of a Lisbon milree in Eng. money, supposing the exchange between London and Amsterdam at 36 sk. 8 gr. per £, and between Amsterdam and Lisbon at 50 gr. for 405 rees or ·405 milrees.

Here, ·405 milrees = 50 gr., and £1 = 36 sk. 8 gr. = 440 gr.;

$$\therefore \quad 1 \text{ milree} = \frac{50}{\cdot 405}\text{ gr., and } 1 \text{ gr.} = £\tfrac{1}{440};$$

$$\therefore \quad 1 \text{ milree} = \frac{50}{\cdot 405}\text{ gr.} = £\frac{50}{\cdot 405} \times \tfrac{1}{440} = 5\text{s. } 7\tfrac{1}{4}\text{d.}$$

2. In the last example, determine the value of £1 in milrees, without taking for granted the result there found.

$$\text{In this case, } £1 = 440 \text{ gr., and } 1 \text{ gr.} = \frac{\cdot 405}{50} \text{ milrees};$$

$$\therefore \quad £1 = 440 \text{ gr.} = 440 \times \frac{\cdot 405}{50} \text{ milrees} = 3{\cdot}564 \text{ milrees.}$$

It will be observed, that the number of milrees in £1 may be found by dividing £1 by 5s. 7¼d.

Now if a London merchant wishes to remit a certain sum of money to Lisbon, it would be most advantageous for him to adopt the circular course of exchange, if the direct exchange between London and Lisbon be less than 3·564 milrees per £.

3. Required the value of a Lisbon milree in Eng. money, supposing the exchange between London and Amsterdam at 432 grotes per £, and between Amsterdam and Lisbon at 48 grotes for 400 rees.

Ans. 5s. 6⅔d.

4. Required, from the last example, the value of £1 in milrees.

Ans. 3·6.

5. Required the arbitrated course of exchange between America and Amsterdam, when the exchange between England and America is at 60 per cent., and between England and Amsterdam at 36 sk. 4 gr. per £.

Here, Eng. £1 = 36 sk. 4 gr. = 436 gr.

Amer. £160 = £100 Eng. = 100 × 436 gr. Amster.

$$\therefore \quad \text{,,} \quad £1 = \frac{100 \times 436}{160}\text{ gr.} = 22 \text{ sk. } 8\tfrac{1}{2} \text{ gr.}$$

that is, 22 sk. 8½ gr. Flemish per pound American.

6. How many francs are equal to a pound American, supposing the exchange between England and America at 50 per cent., and between England and France at 25 fr. 50 c. per £ sterling? *Ans.* 17 fr.

7. Required the value of £1 Eng. money in French francs, supposing the exchange between London and Russia at £4 for 25 rubles, between

Russia and Hamburgh at 55 rubles for 133 marks, between Hamburgh and Spain at 95 marks for 44 dollars, and between Spain and France at 11 dollars for 40 francs.

Here we readily find from the data of question; £1 $= \frac{25}{4}$ rubles; 1 ruble $= \frac{133}{55}$ marks; 1 mark $= \frac{44}{95}$ dollars; 1 dollar $= \frac{40}{11}$;

$$\therefore \ £1 = \tfrac{25}{4} \text{ rubles} = \tfrac{25}{4} \times \tfrac{133}{55} \text{ marks} = \tfrac{25}{4} \times \tfrac{133}{55} \times \tfrac{44}{95} \text{ dollars}$$
$$= \tfrac{25}{4} \times \tfrac{133}{55} \times \tfrac{44}{95} \times \tfrac{40}{11} \text{ francs} = 25\tfrac{5}{11} \text{ francs}.$$

8. Required, from the last example, the value of a franc in English money, without taking for granted the result there found. *Ans.* $9\frac{3}{7}$d.

9. What is the value of £1 in francs, supposing the exchange between London and Amsterdam at 37 sk. 6 gr. per £1, and between Amsterdam and Paris at 37 gr. for 2 francs. *Ans.* $24\frac{12}{37}$ francs.

10. Required, from the last example, the value of a franc in English money without taking the answer there given for granted. *Ans.* $9\frac{13}{15}$d.

11. A London merchant owes a Portuguese merchant £600, whether is it better for the latter to have a direct remittance from London at 5s. 6d. per milree, or a circular remittance through Amsterdam and Paris, the exchange between London and Amsterdam being 37 sk. 4 gr. per £; between Amsterdam and Paris at 38 gr. for 2 francs; and between Paris and Lisbon at 460 rees for 3 francs; allowing $1\frac{1}{2}$ per cent. charge for agency.

For the direct remittance, we have

$$\text{No. milrees} = \frac{600 \times 240}{66} = 2181{\cdot}818.$$

For the circular course of remittance, we have

£100 paid to the agent will only give a remittance of £$98\frac{1}{2}$,

$\therefore$ Value £100 = £$98\frac{1}{2}$

$\therefore$,, £600 = £$98\frac{1}{2} \times 6$ = £591.

From the data of the question we readily find,

$$£1 = 448 \text{ gr.}; \ 1 \text{ gr.} = \tfrac{2}{38} \text{ fr.}; \ 1 \text{ fr.} = \frac{{\cdot}46}{3} \text{ milrees};$$

$$\therefore \ £591 = 591 \times 448 \text{ gr.} = 591 \times 448 \times \tfrac{2}{38} \text{ fr.} = 591 \times 448 \times \tfrac{2}{38}$$
$$\times \frac{{\cdot}46}{3} \text{ milrees} = 2136{\cdot}724 \text{ milrees}.$$

Hence it appears that the direct remittance will be the more advantageous to the Portuguese merchant, for by it he will receive 45·094 milrees more than by the circular course of exchange.

11. Required the same as in the last example when the sum is £400, the direct remittance at 5s. 4d. per milree, exchange between London and Amsterdam at 36 sk. per £, between Amsterdam and Paris at 18 gr. for 1 franc, and between Paris and Lisbon 350 rees for 2 francs, 2 per cent. being allowed for agency.

Ans. The circular remittance is better by 146·4 milrees.

65. *Reduction of foreign weights and measures.*

1. Change 1 mile 40 yds. to French metres. One metre = 1·093633 yards.

1 m. 40 yds. = 1800 yds. = 1800 ÷ 1·093633 metres = 1645·94.

2. Change 3 milimetres French in decimal parts of an inch.

3 milimetres = $\cdot003$ metres = $\cdot003 \times 1\cdot093633$ yds. = $\cdot118112$ in.

3. Change 3 miles 24 yards to French metres. *Ans.* 4849·8902.

4. Change 3·4 metres 2 milimetres to English inches. *Ans.* 133·939.

5. Change 5 kilogrammes French to lbs. One kilo. = 2·20548 lb. Av.

5 kilo. = $5 \times 2\cdot0548$ lbs. Av. = 10·274 lbs. Av.

6. How many grains are there in a gramme French?

1 gramme = ·001 kilogramme = $\cdot001 \times 2\cdot20548$ lbs. Av. = 15·438 gr.

7. How many grammes are in 1 lb. 8 oz. Av.?

$1\frac{1}{2}$ lbs. = $1\frac{1}{2} \div 2\cdot20548$ kilogrammes = 680·12 grammes.

8. Change 5 oz. Troy and 8 oz. Av. to French grammes.

5 oz. Troy = 5×480 gr. = 2400; 8 oz. Av. = $\frac{1}{2}$ of 7000 gr. = 3500;
$\therefore$ 2400 + 3500 = 5900 gr. = $5900 \div 15\cdot438$ grammes = 382·17.

9. Change 2 kilogrammes 10 grammes to grains. *Ans.* 31031·1.

10. Change 5 cwt. to Prussian pounds. One pound = 1·0311 lbs. Av. *Ans.* 543·18.

11. Change 62° Eng. thermometer (Fah.) to French degrees (Centigrade). In the former the temperature of freezing water is 32°, and the boiling point 212°; in the latter the freezing point is zero or 0, and the boiling point 100°.

No. degrees Fah. above the Centi. zero = 62° — 32° = 30°; but
180° Fah. = 100° Centi.; $\therefore$ 30° Fah. = $\frac{1}{6}$ of 100° = $16\frac{2}{3}$° Centi.

12. Change 65° and 75° Fah. to degrees Centi. *Ans.* $18\frac{2}{3}$°, 23·8°

13. Change 20° Centi. to degrees Fah.

100° Centi. = 180° Fah., $\therefore$ 20° Centi. = $\frac{1}{5}$ of 180° = 36° Fah.;
but this gives the degrees above the freezing point,
$\therefore$ the required temp. Fah. = 36° + 32° = 68°.

14. Change 15° and 25° Centi. to degrees Fah. *Ans.* 59°, 77°.

15. The Eng. divide the quadrant of the circle into 90 equal parts, the French into 100, each part being a degree. Convert 55 degrees Eng. into Fr. degrees.

$$90^\circ \text{ Eng.} = 100^\circ \text{ Fr.}; \therefore 55^\circ \text{ Eng.} = \frac{100^\circ \times 55}{90} = 61^\circ\cdot111.$$

16. Change 36° and 40° Eng. to Fr. degrees. *Ans.* 40°, $44\frac{4}{9}$°.

17. Change 9° 40′ Eng. to Fr. degrees.

9° 40′ = $9\frac{40}{60}$ degrees = $9\frac{2}{3}$ degrees Eng.; then 90° Eng. = 100° Fr.,
$\therefore$ 1° Eng. = $\frac{10}{9}$° Fr., $\therefore$ $9\frac{2}{3}$° Eng. = $\frac{10}{9} \times 9\frac{2}{3}$ = 10°·74.

18. Change 8°·24 French degrees into Eng. degrees.

100° Fr. = 90° Eng., $\therefore$ 1° Fr. = $\frac{9}{10}$° Eng.,
$\therefore$ 8°·24 Fr. = $\frac{9}{10}$° $\times$ 8·24 = 7°·416 = 7° 24′ 57″·6.

19. Change 37°·3 and 50° Fr. to Eng. degrees.
Ans. 33° 34′ 12″, 45°.

On the ratio of concrete quantities.

66. When two concrete quantities admit of being expressed in the same unit, their ratio is an abstract quantity.

Thus, $\dfrac{2\text{ ft. }6\text{ in.}}{1\text{ yd. }9\text{ in.}} = \dfrac{30\text{ in.}}{45\text{ in.}} = \frac{30}{45} = \frac{2}{3}$. And $\dfrac{\pounds 1}{23\text{s. }4\text{d.}} = \dfrac{240\text{d.}}{280\text{d.}} = \frac{6}{7}$.

Hence the value of the following arithmetical expression may be readily found.

$$\frac{9\text{ cwt.}}{11\text{ cwt. }1\text{ qr.}} \text{ of } \frac{\pounds 1\ 0\text{s. }10\text{d.}}{14\text{s. }7\text{d.}} \text{ of } 4\text{s. }1\text{d.} = \frac{36\text{ qr.}}{45\text{ qr.}} \times \frac{250\text{d.}}{175\text{d.}} \times 4\text{s. }1\text{d.}$$

$$= \tfrac{36}{45} \times \tfrac{250}{175} \times 4\text{s. }1\text{d.} = \tfrac{8}{7} \times 4\text{s. }1\text{d.} = 4\text{s. }8\text{d.}$$

$$\frac{5\text{ min.}}{1\text{ hour}} \times \tfrac{2}{3} \times \frac{14\text{ lbs.}}{1\text{ qr. }7\text{ lbs.}} \text{ of } 135\text{ ft.} = \frac{5\text{ min.}}{60\text{ min.}} \times \tfrac{2}{3} \times \frac{14\text{ lbs.}}{35\text{ lbs.}} \text{ of } 135\text{ ft.}$$

$$= \tfrac{5}{60} \times \tfrac{2}{3} \times \tfrac{14}{35} \text{ of } 135\text{ ft.} = \tfrac{1}{45} \text{ of } 135\text{ ft.} = 3\text{ ft.}$$

ALLIGATION.

67. This rule is employed in calculating the values of mixtures composed of articles of different qualities.

1. 20 lbs. of sugar, at 7d. per lb., were mixed with 30 lbs. at 5d., and 10 lbs. at 8d.; required the price of the mixture per lb.

Value of the whole $=$ 7d. $\times$ 20 $+$ 5d. $\times$ 30 $+$ 8d. $\times$ 10 $=$ 370d.;
but the No. lbs. in the mixture $= 20 + 30 + 10 = 60$,
$\therefore$ Price of the mixture per lb. $=$ 370d. $\div$ 60 $=$ $6\frac{1}{6}$d.

2. 24 lbs. of tea, at 3s. 4d. per lb., were mixed with 16 lbs. at 5s.; what was the price of the mixture per lb.? *Ans.* 4s.

3. 18 bottles of brandy, at 4s. 6d., were mixed with 36 bottles at 5s. 3d., and with 9 bottles of water; at what price per bottle must the mixture be sold to gain 10 per cent.?

Cost price of the mix. $=$ 4s. 6d. $\times$ 18 $+$ 5s. 3d. $\times$ 36 $=$ 270s.;
but the No. bottles in the mixture $= 18 + 36 + 9 = 63$,
$\therefore$ Cost price mix. per bottle $=$ 270s. $\div$ 63 $=$ $\frac{30}{7}$s.
Selling price of 100s. $=$ 110s.,
$\therefore$,, $\frac{30}{7}$s. $= \frac{110}{100} \times \frac{30}{7}$s. $=$ 4s. $8\frac{4}{7}$d.

4. 24 lbs. of tea, at 4s. 3d. per lb., were mixed with 12 lbs. at 4s. 6d., and 18 lbs. at 5s. 4d.; at what price per lb. must the mixture be sold to gain 25 per cent.? *Ans.* 5s. 10d.

5. 21 bottles of spirits, at 3s., were mixed with 28 bottles at 2s. 6d., and with 7 bottles of water, required the price of the mixture per bottle. *Ans.* 2s. $4\frac{1}{2}$d.

6. How many bottles of water must be mixed with 25 bottles of spirits, at 4s., and 30 bottles at 5s. 6d., so that the mixture may be worth 4s. 5d. per bottle?

Value of the mixture $=$ 4s. $\times$ 25 $+$ 5s. 6d. $\times$ 30 $=$ 265s.;
$\therefore$ No. bottles in the whole $=$ 265s. $\div$ 4s. 5d. $= 265 \div 4\frac{5}{12} = 60$.
$\therefore$ No. bottles of water $= 60 - (25 + 30) = 60 - 55 = 5$.

7. Required the same as in the last example, when the price of the mixture is 4s. 6d. per bottle. *Ans.* $3\frac{8}{9}$ bottles.

8. Required the price of the mixture, per bottle, in example 6, supposing no water to be added. *Ans.* 4s. $9\frac{9}{11}$d.

9. How much water should be mixed with 8 bottles of spirits, at 5s. 6d., so that the mixture may be worth 4s. per bottle? *Ans.* 3 bottles.

10. 8 lbs. of tea, at 5s., were mixed with 6 lbs. of another sort; what must be the price of this tea per lb., so that the mixture may be worth 4s. per lb.?

Cost 1st sort = 5s. $\times$ 8 = 40s.; Cost mix. = 4s. $\times$ 14 = 56s.;
$\therefore$ Cost 6 lbs. of the 2nd sort = 56s. — 40s. = 16s.,
$\therefore$ „ 1 lb. „ = $\frac{1}{6}$ of 16s., = 2s. 8d.

11. 20 lbs. of tea, at 5s. 6d., were mixed with 10 lbs. at 5s., and with 20 lbs. of another sort; what must be the price of this tea per lb., so that the mixture may be worth 5s. per lb.? *Ans.* 4s. 6d.

12. How many lbs. of tea at 10s. per lb. must be mixed with 9 lbs. at 4s., so that the mixture may be worth 6s. a lb.?

The gain by selling the 9 lbs. at 6s. = 6s. $\times$ 9 — 4s. 9d. = 18s.,
The loss by selling 1 lb. of the 10s. tea for 6s. = 10s. — 6s. = 4s.

But in order that there should be neither a gain nor a loss, 4s. multiplied by the no. of lbs. must be equal to 18s.,

$\therefore$ No. lbs. of the 10s. tea = 18s. $\div$ 4s. = 18 $\div$ 4 = $4\frac{1}{2}$.

Questions of this kind may be most simply solved by an equation.

Let x = the No. lbs. at 10s.; then $x + 9$ = the No. lbs. in the mix.;

Value of the mix., in shillings $= 10 \times x + 4 \times 9 = 10\,x + 36$,
also, „ „ $= 6 \times (x + 9) = 6\,x + 54$,
$\therefore \quad 10\,x + 36 = 6\,x + 54$,
Subtracting 36 from each side, $10\,x = 6\,x + 18$,
Subtracting $6\,x$ from each side, $4\,x = 18$,
Dividing each side by 4, $x = 18 \div 4 = 4\frac{1}{2}$.

13. How many lbs. of tea, at 2s. 6d. per lb., must be mixed with 12 lbs. at 5s., so that the mixture may be worth 4s. per lb.? *Ans.* 8.

14. How many lbs. of sugar, at 7d. per lb., must be mixed with 12 lbs. at 5d., and two lbs. at 4d., so that the mixture may be worth 5d. per lb.? *Ans.* 1.

15. How much corn, at 2s. 6d., 3s. 4d., and 3s. 8d. per bushel, must be mixed together, that the mixture may be worth 3s. per bushel?

Taking 1 bushel of each of the first two, and putting x = the number bushels of the last; the number of bushels in the mixture $= 1 + 1 + x = 2 + x$; then

Value of the mixture, in pence $= 30 + 40 + 44\,x = 70 + 44\,x$,
also, „ „ $= 36 \times (2 + x) = 72 + 36\,x$,
$\therefore \quad 70 + 44\,x = 72 + 36\,x$,
$\therefore 44\,x = 2 + 36\,x, \therefore 8\,x = 2, \therefore x = \frac{2}{8} = \frac{1}{4}$.

That is, the quantities are 1, 1, and $\frac{1}{4}$, or multiplying each by 4, to get the relative quantities in whole numbers, they may be written 4, 4, and 1.

16. How much wine, at 6s., and 3s. per bottle, must be mixed together, that the mixture may be worth 4s.? *Ans.* 1 and 2 bottles.

17. How much sugar, at 5d., 6d., and 9d. per lb., must be mixed together, that the mixture may be worth 8d.? *Ans.* 1, 1, and 5 lbs.

18. What quantities of tea, at 3s., and 8s. per lb., must be mixed together, so that 24 lbs. of the mixture may be worth 5s. per lb.?

Let x = the No. lbs. at 8s.; then $24 - x$ = the No. lbs. at 3s.;
$\therefore$ Value of the 2nd kind, in shillings $= 8 \times x = 8\,x$,
also, „ 1st „ $= 3 \times (24 - x) = 72 - 3\,x$,

$\therefore$ Value of the mixture, in shillings $= 8x + 72 - 3x = 5x + 72$;
also, ,, ,, $= 5 \times 24 = 120$,
$\therefore\ 5x + 72 = 120$,
Subtracting 72 from each side, $5x = 48$,
Taking the 5th of each side, $x = \frac{1}{5}$ of $48 = 9\frac{3}{5}$ lbs.
and the number lbs. at 3s. $= 24 - 9\frac{3}{5} = 14\frac{2}{5}$.

Verification. 3s. $\times\ 14\frac{2}{5}$ + 8s. $\times\ 9\frac{3}{5}$ = 5s. $\times$ 24.

19. Required the same as in the last example, when the prices of the two teas are 3s. and 5s., and that of the mixture 4s. *Ans.* 12 lbs. each.

20. How much spirits, at 3s., 6s., and 4s. per bottle, must be mixed together to form a mix. of 9 bottles worth 5s. each? *Ans.* 1, 5, and 3.

Here taking 1 bottle, at 3s., and putting $x =$ the number of bottles at 6s., we get the equation, $3 \times 1 + 6 \times x + 4 \times (9 - 1 - x) = 5 \times 9$.

21. What quantities of spirits, at 14s., 18s., and 15s. per gallon, must be mixed together to form a mixture of 10 gallons, at 16s.?

Ans. 1, $3\frac{2}{3}$, $5\frac{1}{3}$ gallons.

22. How much corn, at 4s., 5s., and 7s. per bushel, must be mixed with 6 bushels, at 4s. 6d., that the mixture may be worth 6s. per bushel?

Taking 1 bushel of each of the first two, and putting $x =$ the number of bushels of the third, the number of bushels in the mixture $= 1 + 1 + x + 6 = 8 + x$; then

Value of the mix., in shillings $= 4 + 5 + 7x + 27 = 36 + 7x$,
also, ,, ,, $= 6 \times (8 + x) = 48 + 6x$,
$\therefore\ 36 + 7x = 48 + 6x$, $\therefore\ x = 12$;

Hence the quantities are 1, 1, 12, and 6 bushels.

Verification. $4 \times 1 + 5 \times 1 + 7 \times 12 + 4\frac{1}{2} \times 6 = 6 \times (1 + 1 + 12 + 6)$.

23. How much sugar, at 5d., 6d., and 8d. per lb., must be mixed with 2 lbs. at $4\frac{1}{2}$d., that the mix. may be worth 7d. per lb.? *Ans.* 1, 1, 8 lbs.

24. How much spirits, at 14s. per gallon, must be added to 4 gallons at 18s., and 6 gallons at 16s., to make the mixture worth 15s. per gal.?

Ans. 18 gallons.

25. How many lbs. of tea at 5s. per lb. must be mixed with 8 lbs. at 3s., so that the mixture may be worth 4s. 4d., per lb.?

Gain from the inferior tea = 4s. 4d. $\times$ 8 − 3s. $\times$ 8 = 10s. 8d.;
but this must be the loss from the superior tea;
loss on 1 lb. of this tea = 5s. − 4s. 4d. = 8d.;
$\therefore$ No. lbs. = 10s. 8d. $\div$ 8d. = 16.

26. A person buys wheat at 46s. and 52s. per qr.; in what proportion must he mix them so as to gain 20 per cent. by selling the mixture at 60s. per qr.?

Here, we must find the cost price of the mixture.

Cost price of 120 = 100; $\therefore$ Cost price of 60 = 50;

that is, the mixture cost 50s. per qr.

Let 1 qr. of the first kind be mixed with x qrs. of the second, then

Value mix. $= 46 + 52x$;
also, ,, $= (1 + x) \times 50 = 50 + 50x$;
$\therefore\ 52x + 46 = 50 + 50x$; subtracting 46 from each side
$52x = 4 + 50x$; subtracting $50x$ from each side
$2x = 4$; $\therefore\ x = 2$;

That is, 2 qrs. of the second kind must be mixed with 1 qr. of the first.

27. In what proportion must he mix them so as to gain 30 p. cent.?
Ans. 1 to $\frac{1}{38}$ or as 38 to 1.

28. If teas at 5s. 4d., 5s., and 4s. 8d. be mixed in equal quantities, and the mixture sold at £36 8s. per cwt., what would be the gain p. c.?

Cost price mix. per lb. $= \frac{1}{3}$ (5s. 4d. + 5s. + 4s. 8d.) = 5s.
Selling price mix. per lb. = £36 8s. ÷ 112 = $6\frac{1}{2}$s.
∴ Gain on 5s. = $6\frac{1}{2}$s. — 5s. = $1\frac{1}{2}$s.
∴ " 100 = $1\frac{1}{2} \times 20 = 30$. *Ans.*

29. Find the gain or loss per cent. when the mixture is sold for £25 4s. per cwt. *Ans.* 10 per cent. loss.

ARITHMETICAL PROBLEMS.—EQUATIONS.

68. All the following problems may be fairly solved by the principles contained in this work.

1. A person after spending $\frac{1}{2}$ and $\frac{1}{3}$ of his money, had 3d. left; what had he at first?

Here, Part spent $= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$; ∴ Part left $= 1 - \frac{5}{6} = \frac{1}{6}$;
but the part left is 3d.; ∴ Value $\frac{1}{6}$ money = 3d.; ∴ the money = 18d.

2. The G. N. Railway stock pay 5 per cent. dividend per annum; how much must I give for them to get $4\frac{1}{2}$ per cent. for my money, allowing $\frac{1}{4}$ per cent. on the original price of the stock for expenses in the transfer?

Price to give $4\frac{1}{2}$ per cent. = £100;
∴ " 5 " = £100 × 5 ÷ $4\frac{1}{2}$ = £$111\frac{1}{9}$;
∴ Price of £100 stock = £$111\frac{1}{9}$ — £$\frac{1}{4}$ = £$110\frac{31}{36}$.

3. The rent of a farm consists of a fixed sum together with the value of a certain number of qrs. of wheat; when wheat is 46s. a qr. the rent is £350, when wheat is 56s. a qr. the rent is £380, what will be the rent when wheat is 71s. a qr.?

Increase of rent due to a rise of 10s. a qr. = £380 — £350 = £30;
" " 15s. a qr. = £30 × $\frac{15}{10}$ = £45;
∴ rent required = £380 + £45 = £425.

4. A person has stock in the 3 per cent. consols, which produces him £21 per annum. He sells out at 90, and invests in a Railway when a £100 share is worth £45. What dividend per cent. per annum ought the Railway to pay so that his income may become £35?

No. of £100 stock = £21 ÷ £3 = 7;
Value of £700 stock = £90 × 7 = £630.
No. £100 Railway shares = 630 ÷ 45 = 14;
∴ 14 × Dividend = £35; ∴ Dividend = £35 ÷ 14 = £$2\frac{1}{2}$.

5. If the wholesale dealer sell to the retailer at 5 per cent. profit, and the retailer sell to the consumer at 20 per cent. profit, what is the per centage of total profit?

100 being taken as the first cost, the cost price to the retailer = 105, and the selling price = 105 + $\frac{1}{5}$ of 105 = 126; ∴ Total profit on 100 = 126 — 100 = 26, that is, the profit will be 26 per cent.

6. What should be the retailer's per centage of profit, when the whole profit on the first cost is 50 per cent.?

Here, as before, the cost price to the retailer is 105.

His selling price $= 100 + 50 = 150$;

$\therefore$ His profit $= 150 - 105 = 45$; but this profit is on 105,

$\therefore$ His profit per cent. $= 45 \times 100 \div 105 = 42\frac{6}{7}$.

7. A wine merchant pays £75 for a cask of wine containing 50 dozen quart bottles, and bottles it off into an equal number of quart and half-pint bottles. How many dozens of each has he, and at what price must he sell it per dozen bottles to gain 20 per cent.?

As the half-pint is $\frac{1}{4}$ of the quart, the number of dozen bottles of each sort $= 50 \div 1\frac{1}{4} = 40$.

Selling price 50 doz. $=$ £75 $+ \frac{1}{5}$ £75 $=$ £90;

$\therefore$ „ 1 doz. $=$ £90 $\div$ 50 $=$ £1 16s.

8. A person mixes a quart of water with a gallon of rum, which cost 14s., and then sells it for 16s. a gallon; with a gallon of brandy which cost 20s. he mixes 3 pints of water, and sells it for 24s. a gallon; what does he gain per cent. on the whole, supposing him to sell two gallons of the neat rum to one of the brandy?

2 gallons of the neat rum gives 2 times $1\frac{1}{4}$ gallons or $2\frac{1}{2}$ gallons of the mixture; and 1 gallon of neat brandy gives $1\frac{3}{8}$ gallons of the mixture;

Profit on 2 gals. rum costing 28s. $=$ 16s. $\times 2\frac{1}{2}$ $-$ 28s. $=$ 12s.;

„ 1 gal. brandy costing 20s. $=$ 24s. $\times 1\frac{3}{8}$ $-$ 20s. $=$ 13s.;

$\therefore$ Profit on 48s. cost price $=$ 12s. $+$ 13s. $=$ 25s.;

$\therefore$ „ 100 „ $= 25 \times 100 \div 48 = 52\frac{1}{12}$.

9. How much ought the price of 3 per cent. consols to fall below par, in order that I may get $3\frac{1}{2}$ per cent. on my money?

Price of stock to give 3 per cent. $=$ £100;

$\therefore$ „ „ $3\frac{1}{2}$ „ $=$ £100 $\times 3 \div 3\frac{1}{2} =$ £$85\frac{5}{7}$;

$\therefore$ Price below par $=$ £100 $-$ £$85\frac{5}{7}$ $=$ £$14\frac{2}{7}$.

10. A tenant holds a farm of 360 acres, subject to a certain tax per acre, and a corn-rent of 300 qrs. of wheat. Now if he pays £810 total rent, when the wheat is 50s. per qr., what is the fixed tax per acre?

Value 300 qrs. wheat at 50s. $=$ 50s. $\times$ 300 $=$ £750;

$\therefore$ The tax on 360 acres $=$ £810 $-$ £750 $=$ £60;

$\therefore$ „ 1 acre $=$ £60 $\div$ 360 $=$ £$\frac{1}{6}$ $=$ 3s. 4d.

11. A person buys 80 sheep for £60; he loses two, and sells 8 for 3s. a piece less than they cost him; at what price per head must he sell the remainder to gain £6 on the whole?

Loss on 10 sheep $=$ 24s.; $\therefore$ Cost of 70 $=$ £60 $-$ 24s. $=$ £58 16s.;

$\therefore$ Selling price 70 sheep $=$ £58 16s $+$ £6 $=$ £64 16s.;

$\therefore$ „ 1 „ $=$ £64 16s. $\div$ 70 $=$ 18s. $6\frac{6}{35}$d.

12. What proportion should a ready money price have to a price for credit of 6 months, allowing interest at 5 per cent. per annum?

Taking 100 as the ready money price; the amount of 100 for 6 months will be $102\frac{1}{2}$; therefore the ratio required will be $100 : 102\frac{1}{2}$, or as $1 : 1\frac{1}{40}$.

13. By selling a horse for £18, a person lost 25 per cent.; what will be his gain per cent., when he sells him for £27?

Here we shall first find the cost price of the horse.

Cost price of £75 = £100;
∴ „ £18 = £100 × 18 ÷ 75 = £24;
∴ Gain on £24 = £27 — £24 = £3;
∴ „ £100 = £3 × 100 ÷ 24 = $12\frac{1}{2}$. *Ans.*

14. How much water must be added to 40 gallons of spirits at 15s., to reduce the price to 12s.?

Price of the whole = 15s. × 40 = 600s.;
but as the mixture must be sold for 12s. per gallon,
The number of gallons of the mixture = 600 ÷ 12 = 50;
∴ Number of gallons of water = 50 — 40 = 10.

15. A person gave for a house a bill of £608 due 4 months hence at 4 per cent., and at once sold it for £663; find his gain per cent.

The interest of £100 for 4 months = $\frac{1}{3}$ £4 = £$1\frac{1}{3}$;
∴ The present value of £$101\frac{1}{3}$ = £100;
∴ „ „ £608 = £100 × 608 ÷ $101\frac{1}{3}$ = £600;
which is the true sum paid for the house;
∴ The gain on £600 = £663 — 600 = £63;
∴ „ £100 = $\frac{1}{6}$ £63 = £$10\frac{1}{2}$. *Ans.*

16. A, B, and C are partners; A receives $\frac{1}{3}$ profits, and B twice as much as C. Find C's capital, A's profit being reduced £20 by a fall of 2 per cent. in the rate of profit.

For every £100 of A's capital his profit is reduced £2; but by this fall his profit is reduced £20; ∴ his capital = £100 × $\frac{20}{2}$ = £1000. But A's capital must be $\frac{1}{3}$ of the whole; therefore the whole capital = £3000; and B and C's capital = £3000 — £1000 = £2000. But B's capital must be double C's, ∴ C's capital = $\frac{1}{3}$ of £2000 = £$666\frac{2}{3}$.

69. In the place of the rule of Position we have here introduced the rule of Equations, by which we suppose x to represent the quantity required in the question, and then proceed, from the data of the question, to form an equality, or equation, expressing the relation between the things given and the thing required; whence, by the known operations or principles of numbers, the value of x, or the thing required, is obtained. This equality, or equation, is usually obtained by finding two independent values for the same thing, and then putting them equal to each other.

1. A horse and a cow were bought for £63; now the horse cost 6 times as much as the cow; required the value of each.

In order to solve this question, let us put the horse into cows. Now the question tells us, that the value of the horse is equal to the value of 6 cows, and that their united value is equal to £63; hence we have,

The value of the cow + 6 times *the value of the cow* = £63.

Here *the value of the cow* is to be added to 6 times *the value of the cow*, which must give us 7 times *the value of the cow;* that is,

7 times *value cow* = £63; taking the 7th of these equals,
Value cow = $\frac{1}{7}$ of £63 = £9;
∴ Value horse = 6 times £9 = £54.

Now, for the sake of conciseness, let us put x for *the value of the cow;* then 6 times x, or 6 x, will be *the value of the horse;* but

The value of the cow + the value of the horse = £63; that is,

$x + 6x =$ £63; adding the x's together,

$7x =$ £63; taking the 7th of these equals,

$x =$ £9, the value of the cow,

and $6x = 6 \times$ £9 = £54, the value of the horse.

2. Divide £54 into three parts, which shall be as 2, 3, 4.

Here, $2 + 3 + 4 = 9$, therefore dividing £54 into 9 equal parts, we have, 1 part = $\frac{1}{9}$ of £54 = £6; ∴ 2 parts = £6 × 2 = £12; 3 parts = £6 × 3 = £18; and 4 parts = £6 × 4 = £24. For the verification of these results, we have, £12 + £18 + £24 = £54.

Let $2x$ be put for the 1st part; then $3x$ will be the 2nd; and $4x$ will be the 3rd; but the sum of all these parts is £54,

∴ $2x + 3x + 4x =$ £54; adding the x's,

$9x =$ £54; taking the 9th of these equals,

$x =$ £6; ∴ $2x =$ £12; $3x =$ £18; and $4x =$ £24.

3. A man bought 30 sheep and 5 cows for £140; now each cow cost 8 times as much as each sheep; what did he pay for each sheep?

Here let us turn the cows into sheep. 1 cow = 8 sheep, ∴ 5 cows = 40 sheep, so that we shall have altogether 70 sheep,

∴ Value 70 sheep = £140; ∴ Value 1 sheep = £2.

Let x = the price of a sheep; then $8x$ = the price of a cow;

∴ price 30 sheep = $30x$; and price 5 cows = 5 times $8x = 40x$;

but, price 30 sheep + price 5 cows = £140; that is,

$30x + 40x =$ £140; adding the x's,

$70x =$ £140; dividing these equals by 70,

$x =$ £2, the price of each sheep.

4. Solve by equations; ques. 41 and 42, Art. 24; and ques. 8, 9, 10, 24, Art. 60.

5. A horse and chaise cost £49; but the horse cost £7 more than the chaise; required the cost of the chaise.

As the horse cost £7 more than the chaise,

Twice the value chaise = £49 — £7 = £42; value chaise = £21.

Let x = the value of the chaise, in pounds; then

$x + 7 =$,, horse, ,, but

Value of the chaise + Value of the horse = £49; that is,

$x + x + 7 = 49$; adding the x's,

$2x + 7 = 49$; subtracting 7 from these equals,

$2x = 42$; taking the half of these equals,

$x =$ £21, the value of the chaise.

6. Solve by equations; ques. 11, 23, Art. 60; and ques. 45, 46, Art. 24.

7. Divide £18 between A and B; so that B shall have $3\frac{1}{2}$ times as much as A.

Supposing A to have 1 share, B will have $3\frac{1}{2}$ shares; then

1 share + $3\frac{1}{2}$ shares = £18; adding the shares,

$4\frac{1}{2}$ shares = £18; multiplying these equals by 2,

9 shares = £36; taking the 9th of these equals,

1 share = £4, A's;

and B's = £18 — £4 = £14.

Let x = A's share; then B's share = $3\frac{1}{2}$ times $x = 3\frac{1}{2}x$;

but the two shares taken together amount to £18,

$\therefore\ x + 3\frac{1}{2}\,x = £18$; adding the x's,
$4\frac{1}{2}\,x = £18$; multiplying these equals by 2,
$9\ x = £36$; taking the 9th of these equals,
$x = £4$, A's share, as before.

8. The fifth of my money added to 4s. is 7s.; how much have I?

$\frac{1}{5}$ the money + 4s. = 7s.; taking 4s. from these equals,
$\frac{1}{5}$ the money = 3s.; taking these equals 5 times,
the money = 3s. × 5 = 15s.

Or putting x for the money, in shillings, we have

$\frac{x}{5} + 4 = 7$; taking 4 from these equals,

$\frac{x}{5} = 3$; taking these equals 5 times,

$x = 3 \times 5 = 15$s., the money.

9. Divide £35 between A and B, so that A's share may be $\frac{3}{4}$ of B's.

Supposing B to have 4 shares; A will have 3 shares, for $\frac{3}{4}$ of 4 is 3; then
4 shares + 3 shares = £35; adding the shares,
7 shares = £35; taking the 7th of these equals,
1 share = £5;
$\therefore$ 4 shares = £5 × 4 = £20, B's; and 3 shares = £5 × 3 = £15, A's.

Let $4\ x$ = B's share; then A's share = $\frac{3}{4}$ of $4\ x = 3\ x$; but
A's share + B's share = £35; that is,
$3\ x + 4\ x = £35$; adding the x's,
$7\ x = £35$; taking the 7th of these equals,
$x = \frac{1}{7}$ of $£35 = £5$;
$\therefore$ A's share $= 3\ x = 3$ times $£5 = £15$; and B's $= 4\ x = £20$.

Or thus. Let x = B's share; then A's = $\frac{3}{4}$ of $x = \frac{3\ x}{4}$;

$\therefore\ x + \frac{3\ x}{4} = £35$; multiplying these equals by 4,

$4\ x + 3\ x = £140$; adding the x's,
$7\ x = £140$; taking the 7th of these equals,
$x = £20$, B's share; and A's share $= \frac{3}{4}\ x = \frac{3}{4}$ of $£20 = £15$.

10. Divide £22 between A, B, and C, so that A's share may be $\frac{1}{2}$ of B's, and C's $\frac{1}{3}$ of B's.

Supposing B to have 6 shares; A will have 3 shares; and C will have 2 shares, because $\frac{1}{3}$ of 6 is 2; then the number of shares will be 11; and $\therefore$ 1 share = $\frac{1}{11}$ of £22 = £2; $\therefore$ 2 shares = £2 × 2 = £4, C's; 3 shares = £2 × 3 = £6, A's; and 6 shares = £2 × 6 = £12, B's. For the verification, we have, £4 + £6 + £12 = £22.

Let $6\ x$ = B's share; A's share $= \frac{1}{2}$ of $6\ x = 3\ x$; and C's share $= \frac{1}{3}$ of $6\ x = 2\ x$; but
A's share + B's share + C's share = £22; that is,
$3\ x + 6\ x + 2\ x = £22$; adding the x's,
$11\ x = £22$; taking the 11th of these equals,
$x = £2$;
$\therefore$ B's share $= 6\ x = 6$ times $£2 = £12$; A's $= 3\ x = £6$;
C's $= 2\ x = £4$.

Or thus. Let x = B's share; A's share = $\frac{1}{2}$ of B's = $\frac{1}{2}$ of $x = \frac{x}{2}$; and C's share = $\frac{1}{3}$ of B's = $\frac{1}{3}$ of $x = \frac{x}{3}$: then

$x + \frac{x}{2} + \frac{x}{3} =$ £22; multiplying these equals by 6,

$6x + 3x + 2x =$ £132; adding the x's,

$11x =$ £132; taking the 11th of these equals,

$x =$ £12, B's share;

$\therefore$ A's share = $\frac{1}{2}x = \frac{1}{2}$ of £12 = £6; and C's share = $\frac{1}{3}x =$ £4.

11. Solve by equations: ques. 15, 16, 18, 19, Art. 60; and ques. 4, 5, 10, 11, Art. 24.

12. To solve ques. 13, Art. 60, by the method of equations.

Let $10x$ = the Captain's share; $5x$ = each Officer's; and $2x$ = each Sailor's;

$\therefore$ 2 Officers' share = 2 times $5x = 10x$; and 7 Sailors' share = 7 times $2x = 14x$;

but the sum of all their shares must be equal to £289,

$\therefore$ $10x + 10x + 14x =$ £289; adding the x's,

$34x =$ £289; dividing these equals by 17,

$2x =$ £17; each Sailor's share;

$\therefore$ $10x =$ £17 $\times 5 =$ £85, the Captain's share;

$\therefore$ $5x = \frac{1}{2}$ of £85 = £42 10s., each Officer's share.

13. Solve by equations; ques. 14, 17, 20, Art. 60.

14. Divide £56 between A, B, C, so that B may have £4 more than A, and C three times as much as B.

Let x = A's share; then B's = $x + 4$; and C's = 3 times $(x + 4) = 3x + 12$:

$\underline{\text{A's share}} + \underline{\text{B's share}} + \text{C's share} = 56$; that is,

$x + \overline{x + 4} + \overline{3x + 12} = 56$; adding the x's, &c.,

$5x + 16 = 56$: taking 16 from these equals,

$5x = 40$; taking the 5th of these equals,

$x =$ £8, A's share;

B's share = $x + 4 =$ £12; and C's = 3 times £12 = £36.

15. A gentleman bought a gig, horse, and harness for £57, the horse cost £9 more than the harness, and the gig twice as much as the horse and harness; required the cost of each. *Ans.* £5, £14, £38.

16. To solve ques. 37, Art. 24, by equations.

Let x = the true time past noon; then

Gain in 1 hour by the watch = $\frac{6}{12}$ min. = $\frac{1}{120}$ hours.

$\therefore$,, x hours ,, $= x$ times $\frac{1}{120} = \frac{x}{120}$,

$\therefore$ *The time by the watch* $= x + \frac{x}{120}$;

but *the time by the watch* is $7\frac{1}{3}$ hours = $\frac{22}{3}$ hours,

$\therefore$ $x + \frac{x}{120} = \frac{22}{3}$; multiplying these equals by 120,

$120x + x = \frac{22}{3} \times 120 = 880$;

$\therefore$ $121x = 880$; $\therefore$ $x = 880 \div 121 =$ 7 h. $16\frac{44}{121}$ min.

17. Solve by equations; ques. 38, Art. 24.

18. To solve ques. 47, Art. 24, and ques. 22, Art. 60, by equations.

1st, Let $x =$ the number of hours required; then

$$\text{No. miles gone by A} = 3\tfrac{3}{4} \times x = \tfrac{15}{4}\,x,$$
$$\text{,,} \qquad \text{B} = 2\tfrac{1}{2} \times x = \tfrac{5}{2}\,x;$$

but the sum of these distances must be 40 miles,

$\therefore\ \frac{15}{4}x + \frac{5}{2}x = 40$; multiplying these equals by 4,

$15x + 10x = 160$; adding the x's,

$25x = 160$; $\therefore\ x = 160 \div 25 = 6\frac{2}{5}$ hours.

2nd, Let $x =$ the number of hours they take to meet; then

No. miles gone by the L. traveller $= 5x$;

,, Y. ,, $= 3x$;

but the sum of these distances is equal to 200 miles,

$\therefore\ 5x + 3x = 200$, $\therefore\ x = \frac{1}{8}$ of $200 = 25$ hours;

$\therefore$ No. miles gone by the Y. traveller $= 25 \times 3 = 75$.

Or thus. Let $x =$ the distance from York, in miles; then

$200 - x =$,, London, ,,

The number of hours for the Y. traveller $= \dfrac{x}{3}$;

,, L. ,, $= \dfrac{200 - x}{5}$;

but the men are the same time in travelling,

$\therefore\ \dfrac{x}{3} = \dfrac{200 - x}{5}$; multiplying by 5 and 3,

$5x = 3 \times (200 - x)$; completing the multiplication,

$5x = 600 - 3x$; adding $3x$ to these equals,

$8x = 600$; $\therefore\ x = 75$ miles.

19. Solve by equations; ques. 49, Art. 24; and ques. 41, Art. 39.

20. To solve ques. 74, Art. 39, by equations.

Let $x =$ the number of dollars; then we obviously have

$4\frac{1}{2} \times x = 7\frac{1}{2} \times 42$; multiplying these equals by 2,

$9x = 15 \times 42$; $\therefore\ x = 15 \times 42 \div 9 = 70$.

21. Solve by equations; ques. 75 and 78, Art. 39.

22. To solve ques. 87, Art. 39, by equations.

Let $x =$ the number of hours the fast train is travelling; then

$x + 2 =$,, slow ,,

No. miles gone by the fast train $= 30 \times x$

,, slow ,, $= 20 \times (x + 2) = 20x + 40$;

but these distances are the same,

$\therefore\ 30x = 20x + 40$; subtracting $20x$ from these equals,

$10x = 40$; $\therefore\ x = 4$, the number of hours.

23. Solve by equations; ques. 88, 89, Art. 39.

24. To solve ques. 90, Art. 39, by equations.

Let $x =$ the number of minutes past IV. Now when the short pointer is at IV the long pointer is at XII, that is, the former is 20 minute spaces in advance of the latter; therefore when the pointers are together the long pointer will have moved over x minute space, and the short one only $x - 20$; but the former is 12 times the latter, because the long pointer moves 12 times as fast as the short one,

$\therefore$ 12 times $(x - 20) = x$;

$\therefore$ $12\,x - 240 = x$; adding 240 to these equals,

$12\,x = x + 240$; taking x from these equals,

$11\,x = 240$; $\therefore$ $x = \frac{1}{11}$ of $240 = 21\frac{9}{11}$ min.

$\therefore$ The time will be $21\frac{9}{11}$ min. past 4.

25. Solve by equations; ques. 91, Art. 39.

26. To solve ques. 5, Art. 49, by equations.

Let $x =$ the gain per cent. per annum; then

The gain $=$ £$4000 \times 1\frac{5}{8} \times x \div 100 =$ £$60\,x$; but *the gain* is £650;

$\therefore$ $60\,x = 650$; $\therefore$ $x = 10\frac{5}{6}$.

27. Solve by equations; ques. 1, 2, 3, 4, Art. 49.

28. To solve ques. 2, Art. 50, by equations.

Let $x =$ the principle in pounds; then by the rule of interest,

$$\text{Interest, in £} = \frac{x \times 5 \times 146}{100 \times 365}; \text{ but the interest is £40;}$$

$$\therefore\ \frac{x \times 5 \times 146}{100 \times 365} = 40;\ \therefore\ x = \frac{40 \times 100 \times 365}{5 \times 146} = \text{£}2000.$$

29. Solve by equations, ques. 1, 3, Art. 50.

30. To solve ques. 1, Art. 51 by equations.

Let $x =$ the number of days; then *the interest* $=$ £$\frac{500 \times 4 \times x}{100 \times 365}$;

but *the interest* $=$ £516 $-$ £500 $=$ £16;

$$\therefore\ \frac{500 \times 4 \times x}{100 \times 365} = 16;\ \therefore\ x = \frac{16 \times 365}{5 \times 4} = 292 \text{ days.}$$

31. Solve by equations; ques. 2, 3, Art. 51.

32. To solve ques. 6 and 8, Art. 54, by equations.

1st. Let $x =$ the present worth in £. Here the present worth added to its interest for the given time must be equal to £350.

$$\text{Interest on } x \text{ pounds} = \frac{x \times 3 \times 511}{100 \times 365} = \frac{x \times 21}{500},$$

$$\therefore\ x + \frac{x \times 21}{500} = 350; \text{ multiplying these equals by 500,}$$

$500\,x + 21\,x = 175000$; adding the x's,

$521\,x = 175000$; dividing these equals by 521,

$x = 175000 \div 521 =$ £335 17s. 10d.

2nd. Let $x =$ the sum in £. In this case the sum added to its interest for a year must produce £630; hence, we have,

$x + x \times 5 \div 100 =$ £630; multiplying these equals by 100,

$100\,x + 5\,x =$ £63000; adding the x's,

$105\,x =$ £63000; $\therefore$ $x =$ £$63000 \div 105 =$ £600.

33. Solve by equations any of the questions in Art. 54.

34. A person after paying an income-tax of 7d. in the £, has £233 left; what was his gross income?

Here every 19s. 5d. left corresponds with £1 income;

$\therefore$ No. pounds income $=$ £233 $\div$ 19s. 5d. $=$ £240.

Or thus. Let $x =$ his income in £; then the income-tax $= x \times \frac{7}{240}$. But his income will be equal to the money left added to the sum paid for income-tax,

$\therefore\ x = 233 + x \times \frac{7}{240}$; multiplying these equals by 240,
$240\,x = 55920 + 7\,x$; taking $7\,x$ from these equals,
$233\,x = 55920$; $\therefore\ x = 55920 \div 233 =$ £240.

35. A ship worth £3600 being lost, of which A's share was £200 more than B's, and C's half as much as A's and B's taken together; find the loss which each will sustain if she be insured for £3000.

Let $x =$ B's share of the ship; then A's $= x + 200$;

$$\text{and C's} = \tfrac{1}{2}(\text{A's} + \text{B's}) = \tfrac{1}{2}(x + x + 200) = \tfrac{1}{2}(2\,x + 200) = x + 100.$$

But the sum of their shares must be equal to £3600,

$\therefore\ x + \overline{x + 200} + \overline{x + 100} = 3600$; adding the x's, &c.,
$3\,x + 300 = 3600$; taking 300 from these equals,
$3\,x = 3300$; $\therefore\ x = 1100$, B's share;
$\therefore$ A's share $= 1100 + 200 = 1300$, and C's $= 1100 + 100 = 1200$

Now dividing the loss, £600, in the ratio of their shares (See Art. 60), we find, A's loss $=$ £600 $\times \frac{1300}{3600} =$ £$216\frac{2}{3}$; and so on to the others.

36. A grocer sells 500 lbs. of tea at a profit of 6 per cent., and 300 lbs. at a profit of 10 per cent.; if he had sold the whole at a uniform profit of 8 per cent. he would have received 12s. more than he actually got; what was the cost price of the tea per lb.?

1st. Profit at 1s. per lb. cost $= 500 \times \frac{6}{100} + 300 \times \frac{10}{100} = 60$s.;
2nd. ,, ,, $= 800 \times \frac{8}{100} = 64$s.;
$\therefore$ Excess of profit, the tea being 1s. per lb. $= 64 - 60 = 4$s.;
but the excess of profit, by the ques., is 12s.,
$\therefore$ Price of the tea per lb. in shillings $= 12 \div 4 = 3$s.

Or thus. Let $x =$ the cost price in shillings; then $500\,x =$ cost of 500 lbs., &c.

1st. Profit $= 500\,x \times \frac{6}{100} + 300\,x \times \frac{10}{100} = 30\,x + 30\,x = 60\,x$;
2nd. ,, $= 800\,x \times \frac{8}{100} = 64\,x$.
But, by the question, the latter is 12s. more than the former,
$\therefore\ 64\,x - 60\,x = 12$; subtracting the x's,
$4\,x = 12$; $\therefore\ x = 3$s. *Ans.*

37. A gentleman dying leaves property worth £2100 among 3 sons and 2 daughters, directing that the sons shall have alike $\frac{1}{4}$ more than the elder daughter, who should have £200 more than the younger. How much did each get?

Let $4\,x =$ the elder daughter's portion; then the younger one's $= 4\,x - 200$; each son's portion $= 4\,x + x = 5\,x$; and 3 sons' portion $= 3$ times $5\,x = 15\,x$.

But the sum of all their portions must make up £2100,
$\therefore\ 15\,x + 4\,x + 4\,x - 200 = 2100$; adding the x's,
$23\,x - 200 = 2100$; adding 200 to these equals,
$23\,x = 2300$; $\therefore\ x =$ £100;

Elder daughter's portion $= 4\,x =$ £400; younger daughter's portion $=$ £200; and each son's portion $= 5\,x =$ £500.

38. A person buys 60 tons of coals, and gains £12 by selling them at 2s. 3d. per sack. If he had sold them at 1s. 11d. per sack he would have lost £4. Find the cost price per ton, and the weight of a sack.

Let $x =$ the number of sacks in the whole; 1st selling price in shillings $= 2\frac{1}{4} \times x$; 2nd selling price $= 1\frac{11}{12} \times x$. Now we shall get two independent values for the cost price of the whole in shillings.

1st. Cost price of the whole $= 2\frac{1}{4} \times x - 12 \times 20 = 2\frac{1}{4}\,x - 240$
2nd. „ „ $= 1\frac{11}{12} \times x + 4 \times 20 = 1\frac{11}{12}\,x + 80$

$\therefore\ 2\frac{1}{4}\,x - 240 = 1\frac{11}{12}\,x + 80$; multiplying these equals by 12,
$27\,x - 2880 = 23\,x + 960$; adding 2880 to these equals,
$27\,x = 23\,x + 3840$; subtracting $23\,x$ from these equals,
$4\,x = 3840$; $\therefore\ x = 960$, number of sacks.
$\therefore$ Weight of each sack $= 60$ tons $\div 960 = 1\frac{1}{4}$ cwt.
Cost price 60 tons in s. $= 2\frac{1}{4} \times 960 - 240 = 1920$
$\therefore$ „ 1 ton „ $= 1920 \div 60 = 32$s.

39. Divide £660 among A, B, C; so that A's share : B's share : : 3 : 2; and B's share : C's share : : 5 : 4.

Here, 3 to 2 is the same as 1 to $\frac{2}{3}$, that is, B's share $= \frac{2}{3}$ of A's share; 5 to 4 is the same as 1 to $\frac{4}{5}$, that is, C's share $= \frac{4}{5}$ of B's. Now let x = A's share in £, then B's share $= \frac{2}{3}\,x$, and C's share $= \frac{4}{5}$ of $\frac{2}{3}\,x = \frac{8}{15}\,x$. But the sum of these shares must make up £660,

$\therefore\ x + \frac{2}{3}\,x + \frac{8}{15}\,x = 660$; multiplying these equals by 15,
$15\,x + 10\,x + 8\,x = 660 \times 15 = 9900$; adding the x's,
$33\,x = 9900$; $\therefore\ x = 9900 \div 33 =$ £300, A's;
$\therefore$ B's share $= \frac{2}{3}\,x =$ £200; and C's $= \frac{8}{15}\,x =$ £160.

40. To solve ques. 21 and 24, Art. 59, by equations.

1st. Let x = the cost price in £. Here the cost price added to the gain upon it must be equal to the selling price or £169.

Gain on x pounds $= x \times 30 \div 100 = \frac{3}{10}\,x$,
$\therefore\ x + \frac{3}{10}\,x =$ £169; multiplying these equals by 10,
$10\,x + 3\,x =$ £1690; $\therefore\ x =$ £130.

2nd. Let x = the selling price in £, to gain 25 per cent.; then

$$\textit{Cost price} = \frac{100 \times x}{125}; \text{ also } \textit{Cost price} = \frac{100 \times 33}{110};$$

$$\therefore\ \frac{100 \times x}{125} = \frac{100 \times 33}{110};\ \therefore\ x = \frac{33 \times 125}{110} = £37\tfrac{1}{2}.$$

41. Solve by equations; ques. 20, 22, 23, Art. 59.

42. A person wishing to serve some beggars, found, that if he gave each 3d. he would have 4d. left; but if he gave each 5d. he would want 8d.; how many beggars were there?

Put x for the number of beggars. In order to solve this question, we shall find two separate values for the number of pence the person had, in terms of x, and then we shall put these two values for the same thing equal to each other.

No. pence that would be 1st given away $= 3\,x$;
but the number of pence the person had is 4 more than this,
$\therefore$ *Number of pence the person had* $= 3\,x + 4$.
No. pence given away in the 2nd case $= 5\,x$;
but the number of pence the person had is 8 less than this,
$\therefore$ *No. pence the person had* $= 5\,x - 8$,
$\therefore\ 5\,x - 8 = 3\,x + 4$; adding 8 to these equals,
$5\,x = 3\,x + 12$; taking $3\,x$ from these equals,
$2\,x = 12$, $\therefore\ x = 6$.

43. I want to divide some nuts among a certain number of boys. If I were to give 6 nuts to each boy, I should have 4 nuts to spare, and if

I were to give 8 nuts to each boy, I should have 14 nuts too few. How many boys are there? *Ans.* 9.

44. Divide a rod of 10 ft. long into two parts, so that the one part may be 4 ft. longer than the other. *Ans.* 3 and 7 ft.

45. A man is 5 years older than his wife, but 10 years hence her age will be $\frac{7}{8}$ of his age. Required their ages.

Let $x =$ the wife's present age; then $x + 5 =$ the man's age;
the wife's age 10 years hence $= x + 10$; the man's 10 years hence $= x + 15$; but the former is $\frac{7}{8}$ of the latter,

$\therefore\ x + 10 = \frac{7}{8}(x + 15)$; multiplying these equals by 8,
$8(x + 10) = 7(x + 15)$; completing the multiplication,
$8x + 80 = 7x + 105$; taking $7x$ from these equals,
$x + 80 = 105$; $\therefore\ x = 25$, the wife's age;
and the man's age $= x + 5 = 30$.

46. John is 2 years older than Tom, but 3 years hence Tom's age will be $\frac{5}{6}$ of John's. Required their ages. *Ans.* 7, 9.

47. A man is 28 years older than his son, and their united ages is 52. Required the son's age. *Ans.* 12.

48. If 20 be added to a certain number, the sum will be 6 times the number itself. Required the number. *Ans.* 4.

49. A man spends £30 of his yearly income in house rent, and $\frac{2}{3}$ the remainder in general expenditure, and at the end of the year he lays by £20. What is his income?

Here since he spends $\frac{2}{3}$ the remainder, $\frac{1}{3}$ of the remainder is what he lays by; but this is £20,

$\therefore\ \frac{1}{3}$ the remainder $=$ £20; $\therefore$ the remainder $=$ £60;
$\therefore$ His income $=$ £60 $+$ £30 $=$ £90.

Or thus. Let $x =$ the man's income in £; then the expenditure $= \frac{2}{3}(x - 30)$.

The income, in £ $=$ Rent $+$ Expenditure $+$ Savings
$= 30 + \frac{2}{3}(x - 30) + 20 = 50 + \frac{2}{3}(x - 30)$;
but the income is x pounds,

$\therefore\ x = 50 + \frac{2}{3}(x - 30)$; multiplying these equals by 3,
$3x = 150 + 2(x - 30)$; completing the multiplication,
$3x = 150 + 2x - 60$; $\therefore\ x =$ £90.

50. To solve ques. 12, Art. 24, by equations.

Let $x =$ the sum; then sum 1st spent $= \frac{1}{3}x$;
$\therefore$ Sum then left $= x - \frac{1}{3}x = \frac{2}{3}x$.
Sum next spent $= \frac{3}{4}$ of $\frac{2}{3}x = \frac{6}{12}x = \frac{1}{2}x$;
$\therefore$ Sum now left $= \frac{2}{3}x - \frac{1}{2}x = \frac{4}{6}x - \frac{3}{6}x = \frac{1}{6}x$;
but this sum now left is equal to $2\frac{1}{3}$ shillings,
$\therefore\ \frac{1}{6}x = 2\frac{1}{3}$s., $\therefore\ x = 6$ times $2\frac{1}{3}$s. $=$ 14s.

51. A gave $\frac{2}{5}$ of his money to B, and $\frac{1}{2}$ of the remainder to C, and then found that he had 9d. left; what had he at first? *Ans.* 30d.

52. Solve ques. 13, Art. 24, by equations.

53. How much tea at 6s. per lb. must be mixed with 10 lbs. at 3s. per lb., so that the mixture may be worth 4s. per lb.?

Let $x =$ the no. lbs. at 6s.; then $x + 10 =$ no. lbs. in the mixture;
Value mixture, in s. $= 6 \times x + 3 \times 10 = 6x + 30$,
also, ,, ,, $= 4 \times (x + 10) = 4x + 40$;

$\therefore \quad 6x + 30 = 4x + 40$; taking $4x$ from these equals,
$2x + 30 = 40$; taking 30 from these equals,
$2x = 10$; $\therefore x = 5$, the no. lbs.

54. Solve ques. 6, 10, Art. 67, by equations.

55. How many crowns and two-shilling pieces will pay a bill of £26, the number of pieces, of both kinds, being 140?

Let x = the no. of crowns; then $140 - x$ = the no. of 2s. pieces.

Value of the crowns, in s. $= 5x$,
Value of the 2s. pieces, in s. $= 2(140 - x) = 280 - 2x$;

but the value of the two together will be £26 or 520 shillings,

$\therefore \quad 5x + 280 - 2x = 520$; taking the $2x$ from the $5x$,
$3x + 280 = 520$; taking 280 from these equals,
$3x = 240$; $\therefore x = 80$, the no. crowns;
and the no. of 2s. pieces $= 140 - 80 = 60$.

56. How many crowns and shillings will pay a bill of £2 10s., the number of pieces, of both kinds, being 22? *Ans.* 7, 15.

70. *Operation of subtraction.* Before proceeding with these problems, it is requisite that we should explain the following fundamental operation of subtraction. From £9 let it be required to subtract £5 less by £3. Now this is obviously the same as subtracting £2 from £9, but we want to show how the operation of subtraction effects the signs of the quantities. Taking £5 from £9 there will be £4 left, but this result is too little, for we have taken away £3 too much; therefore the true result will be obtained by adding £3 to the £4; this operation is expressed as follows: £9 — (£5 — £3) = £9 — £5 + £3; where we change the signs of the quantities to be subtracted. In like manner,

$$9x - (5x - 3) = 4x + 3;$$

here taking $5x$ from $9x$ leaves $4x$; but we have taken away 3 too much; therefore the true result is obtained by adding 3 to the $4x$. In like manner,

$$5x - (8 - 4x) = 5x - 8 + 4x = 9x - 8;$$

here taking 8 from $5x$ gives $5x - 8$; but we have taken away $4x$ too much; therefore the true result is obtained by adding $4x$ to $5x - 8$.

1. To solve ques. 51, Art. 24, by equations.

Let x = the no. working days; then $24 - x$ = the no. idle days.

Sum due on the working days, in s. $= 2x$.
Sum charged for the idle days, ,, $= \frac{3}{4}(24 - x)$.

Now the sum due to the labourer will be found by subtracting the latter from the former; but the sum due to him is 37s.,

$\therefore \quad 2x - \frac{3}{4}(24 - x) = 37$; mult. these equals by 4,
$8x - 3(24 - x) = 148$;

completing the multiplication and then subtracting the result,

$$8x - 72 + 3x = 148; \ \therefore x = 20.$$

2. Solve ques. 52, Art. 24, by equations.

3. A grocer bought 21 lbs. of tea; after having sold 14 lbs. at 10 p.c. profit, he sold the remainder at 5 p.c. profit, and thereby gained 7s. on the whole; required the cost price per lb.

Gain on 1s. per lb. cost price $= \frac{1}{10} \times 14 + \frac{1}{20} \times 7 = \frac{7}{4}$s.;
but the whole gain is 7s.,

$\therefore$ Cost price per lb. in shillings $= 7 \div \frac{7}{4} = 4$s. *Ans.*

By equations. Let $x =$ the cost price in shillings; then

$$\text{Gain on 14 lbs.} = \frac{x}{10} \times 14 = \frac{7\,x}{5}; \text{ gain on 7 lbs.} = \frac{7\,x}{20};$$

$$\therefore \quad \frac{7\,x}{5} + \frac{7\,x}{20} = 7; \text{ mult. these equals by 20,}$$

$$28\,x + 7\,x = 140; \therefore x = 4\text{s.}$$

4. Required the same as in the last example, when the whole gain is 5s. 10d. *Ans.* 3s. 4d.

5. A man travelled a journey at the rate of 5 miles an hour, and returned the distance at the rate of 4 miles an hour. He took 18 hours in going and returning. Required the total distance gone over.

Let $x =$ the distance each way, in miles; then

$$\frac{x}{5} = \text{no. hours in going; and } \frac{x}{4} = \text{no. hours in returning;}$$

$$\therefore \quad \frac{x}{5} + \frac{x}{4} = 18; \text{ multiplying these equals by 20,}$$

$$4\,x + 5\,x = 360; \therefore 2\,x = 80 \text{ miles, the total distance.}$$

6. Required the total distance, supposing the rates of travelling to be 4 and 3 miles respectively. *Ans.* $61\frac{5}{7}$ miles.

7. From a cask of wine, a sixth had leaked away, 30 gallons were drawn, and then it was one-third full; how much did it hold?

Here, $\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$, $\therefore$ the part drawn $= 1 - \frac{1}{2} = \frac{1}{2}$; but there were 30 gallons drawn, $\therefore \frac{1}{2}$ part $=$ 30 gals., and $\therefore$ the whole $=$ 60 gals.

By equations. Let $x =$ the no. gals. in the cask; then

$$\textit{no. gals. left} = x - \frac{x}{6} - 30; \text{ but } \textit{no. gals. left} = \frac{x}{3};$$

$$\therefore \quad x - \frac{x}{6} - 30 = \frac{x}{3}; \text{ multiplying by 6,}$$

$$6\,x - x - 180 = 2\,x; \text{ adding 180 to these equals, \&c.,}$$

$$5\,x = 2\,x + 180; \text{ taking } 2\,x \text{ from these equals,}$$

$$3\,x = 180; \therefore x = 60, \text{ the number gallons.}$$

8. A gamester lost $\frac{1}{5}$ of his money, and afterwards lost £2, and then he had $\frac{2}{3}$ of his money remaining. How much had he at first?

Ans. £15.

9. Divide £34 between A, B, and C, so that B may have £2 for A's £3, and C £7 for A's £6.

Suppose A to have 6 shares, then B will have $\frac{2}{3}$ of 6 shares, or 4 shares, and C will have $\frac{7}{6}$ of 6 shares, or 7 shares. Adding these shares together we shall have, the value of 17 shares $=$ £34; $\therefore$ 1 share $=$ £2; $\therefore$ 6 shares $=$ £12, A's; 4 shares $=$ £8, B's; and 7 shares $=$ £14, C's.

$$\text{Let } x = \text{A's share; then B's} = \tfrac{2}{3}\,x; \text{ C's} = \tfrac{7}{6}\,x;$$

$$\therefore \quad x + \tfrac{2}{3}\,x + \tfrac{7}{6}\,x = \text{£}34; \text{ multiplying these equals by 6,}$$

$$6\,x + 4\,x + 7\,x = \text{£}204; \text{ collecting the } x\text{'s.}$$

$$17\,x = \text{£}204; \therefore x = \text{£}12, \text{ A's share;}$$

$$\text{B's share} = \tfrac{2}{3} \text{ of £}12 = \text{£}8; \text{ and C's} = \tfrac{7}{6} \text{ of £}12 = \text{£}14.$$

10. A person travelled altogether 60 miles; he went 4 miles on foot to 6 miles by water, and 5 on horseback to 3 by water; how many miles did he travel by water? *Ans.* 18.

11. Find a number such that if $\frac{2}{3}$ of it be taken from 14, and $\frac{1}{4}$ of the remainder from $\frac{1}{2}$ of the original number, 12 times this last remainder shall be equal to the original number.

Here putting x for the number, we readily find the following equation:

$12 \left\{ \frac{1}{2} x - \frac{1}{4} (14 - \frac{2}{3} x) \right\} = x$; performing the multiplication by 12,

$6 x - 3 (14 - \frac{2}{3} x) = x$; multiplying and then subtracting,

$6 x - 42 + 2 x = x$; $\therefore 7 x = 42$; $\therefore x = 6$.

12. Divide 30 into two parts, so that 3 times the less shall exceed $\frac{1}{3}$ of the greater by 10. *Ans.* 6, 24.

13. Find two numbers, whose sum is 24, and quotient 5.

As the quotient is 5, one of the numbers must be 5 times the other. Therefore dividing 24 into 6 equal parts, one part = 4, and 5 parts = 20, that is, the less will be 4 and the greater 20.

Let x = the less, then $24 - x$ = the greater;

$\therefore \frac{24 - x}{x} = 5$; multiplying these equals by x,

$24 - x = 5 x$, or $5 x = 24 - x$; $\therefore 6 x = 24$;

$\therefore x = 4$, the less; and the greater $= 24 - 4 = 20$.

14. Find two numbers, whose difference is 9, and quotient 4. *Ans.* 3, 12.

15. To find a number to which if its half and its fourth parts be added, the sum shall be equal to the square of that number.

Let x = the number; then its square $= x \times x$, or x^2;

$\therefore x \times x = x + \frac{1}{2} x + \frac{1}{4} x$; dividing these equals by x,

$x = 1 + \frac{1}{2} + \frac{1}{4} = 1\frac{3}{4}$.

16. A person being asked his age, replied that $\frac{1}{4}$ of his age multiplied by $\frac{1}{6}$ of his age would produce his age. How old was he? *Ans.* 24.

17. Divide 14 into two parts, such that if the one be divided by 2, and the other by 3, the sum of the quotients shall be 6. *Ans.* 8, 6.

18. A horse was sold at a loss for £30; but if it had been sold for £40, the gain would have been $\frac{2}{3}$ of the former loss; required its value.

Let x = the value in £; then the loss $= x - 30$; the gain $= 40 - x$; but, by the question, $\frac{2}{3}$ of the former is equal to the latter,

$\therefore \frac{2}{3} (x - 30) = 40 - x$; multiplying these equals by 3,

$2 (x - 30) = 120 - 3 x$; performing the multiplication,

$2 x - 60 = 120 - 3 x$; $\therefore 5 x = 180$; $\therefore x =$ £36.

19. An article was sold at a loss for 4s.; but if it had been sold for 5s., the gain would have been $\frac{3}{4}$ of the former loss; required its cost price. *Ans.* 4s. $6\frac{6}{7}$d.

20. A and B have the same income; A lays by $\frac{1}{8}$ of his income; but B, by expending £14 per annum more than A, at the end of 3 years is £12 in debt; required their income.

B spends £4 per annum more than his income; but as B spends £14 per annum more than A, $\therefore$ A spends £10 less than his income, and thereby saves $\frac{1}{8}$ of his income; $\therefore \frac{1}{8}$ of the income = £10; and $\therefore$ the income = £80.

Let $x =$ the income in £; then A's expenditure in 1 year $= x - \frac{1}{8} x = \frac{7}{8} x$; but, by the question, B expends £14 in the year more than A;

$\therefore$ B's *expenditure in* 3 *years* $= 3\left(\frac{7}{8} x + 14\right) = \frac{21}{8} x + 42$;
also, B's ,, $=$ £12 more than his income $= 3x + 12$;

$\therefore \quad 3x + 12 = \frac{21}{8} x + 42$; taking 12 from these equals,
$3x = \frac{21}{8} x + 30$; multiplying these equals by 8,
$24x = 21x + 240$; $\therefore 3x = 240$; $\therefore x =$ £80.

21. Required the same as in the last example, supposing A to lay by $\frac{1}{4}$ of his income. *Ans.* £40.

22. Solve by equations, ques. 35, Art. 59.

Let $x =$ the number of each sort; then $2x =$ the total number;
Cost price, in pence $= \frac{1}{2} x + \frac{1}{4} x = \frac{3}{4} x$;
Selling ,, ,, $= \frac{2}{5} \times 2x = \frac{4}{5} x$;
$\therefore$ Gain $= \frac{4}{5} x - \frac{3}{4} x$; but the gain is 12d.;
$\therefore \frac{4}{5} x - \frac{3}{4} x = 12$; multiplying these equals by 20,
$16x - 15x = 240$; $\therefore x = 240$;

23. Solve by equations, ques. 36, 31, 32, 33, 34, Art. 59.

24. A field was sown with wheat at 34s. per boll, and produced 8 returns; the crop was sold at 30s. per boll and, after paying for the seed, £206 were left; how many bolls were sown? *Ans.* 20.

25. To solve ques. 54, Art. 40, by the method of equations.

Let $5x =$ number steps taken by the pol. to overtake the boy;
$6x =$,, ,, boy;
Distance gone by the pol., in feet $= 2 \times 5x = 10x$;
,, ,, boy, ,, $= \frac{2}{3}$ of $2 \times 6x = 8x$;
but the former distance must exceed the latter by 160,
$\therefore \quad 10x - 8x = 160$; $\therefore 2x = 160$; $\therefore 10x = 800$ ft.

26. Solve by equations, ques. 55, Art. 40.

27. To solve ques. 27, Art. 60, by the method of equations.

Let $x =$ no. lbs. copper added; then $64 + x =$ no lbs. in the whole;
wt. of tin $= \frac{4}{13}(64 + x)$; also, wt. of tin $= \frac{3}{8}$ of 64 lbs. $= 24$ lbs.;
$\therefore \quad \frac{4}{13}(64 + x) = 24$; $\therefore x = 14$ lbs.

28. Solve by equations, ques. 28, Art. 60.

29. A piece of ground is to be enclosed by posts. When they are set 2 feet apart, there are 30 too few, but when they are set 3 feet apart there are 20 too many; what is the circuit of the ground?

Let $x =$ the circuit of the ground in feet; then
number posts, 2 feet apart, to go round the ground $= \frac{1}{2} x$;
,, 3 feet apart, ,, ,, $= \frac{1}{3} x$;
but the number of posts is 30 less than the former, and 20 greater than the latter; that is; *no. posts* $= \frac{1}{2} x - 30$; also, *no. posts* $= \frac{1}{3} x + 20$;
$\therefore \quad \frac{1}{2} x - 30 = \frac{1}{3} x + 20$; multiplying these equals by 6,
$3x - 180 = 2x + 120$; $\therefore x = 300$ feet.

30. Required the circuit of the ground, supposing 50 too few in the first case, and 60 too many in the other case. *Ans.* 660 feet.

31. The tail of a fish weighed 4 lbs., the body as much as the tail and half the head, and the head as much as the tail and one-fifth of the body; what was the weight of the fish?

Here, it will be most convenient to put $x =$ the weight of the head;

then the weight of the body $= 4 + \frac{1}{2} x$; but, by the question, weight head $=$ weight tail $+ \frac{1}{3}$ weight body, that is,

$$x = 4 + \tfrac{1}{3}(4 + \tfrac{1}{2} x); \text{ multiplying these equals by 5,}$$
$$5x = 20 + 4 + \tfrac{1}{2} x; \; x = 5\tfrac{1}{3} \text{ lbs., weight head;}$$
$$\text{weight of the body} = 4 + \tfrac{1}{2} \text{ of } 5\tfrac{1}{3} = 6\tfrac{2}{3} \text{ lbs.;}$$
$$\text{and weight of the whole fish} = 4 + 5\tfrac{1}{3} + 6\tfrac{2}{3} = 16 \text{ lbs.}$$

32. What would be the weight of the fish, supposing the tail to weigh 5 lbs.? *Ans.* 20 lbs.

33. Seven years ago my age was 4 times my son's, and 7 years hence my age will be double his age. Required our ages.

Let $x =$ the son's age 7 years ago; then
$4x =$ the father's age 7 years ago;

$x + 7$, and $4x + 7$, will be their present ages; and $x + 14$, and $4x + 14$, will be their ages 7 years hence; but, by the question, the latter is double the former,

$$\therefore\; 4x + 14 = 2(x + 14); \; \therefore x = 7;$$
$$\therefore \text{ son's pres. age} = x + 7 = 14; \text{ father's pres. age} = 4x + 7 = 35.$$

34. Two years ago my age was 4 times my son's, and 2 years hence my age will be 3 times his age. Required our ages. *Ans.* 10, 34.

35. The paving of a square court, at 2s. the square yard, cost as much as the enclosing of it at 16s. the lineal yard. Find the side of the square.

Let $x =$ the number yards in the side; then
$x \times x$ or $x^2 =$ the number square yards in the court:
$4x =$ the number lineal yards in the enclosure;
$\therefore$ Cost of the pavement, in shillings $= 2x^2$; and
Cost of the enclosure, in shillings $= 16 \times 4x = 64x$;
$\therefore\; 2x^2 = 64x$; dividing these eqnals by x,
$2x = 64$; $\therefore x = 32$ yards, the side.

36. What would be the side of square, supposing the pavement to cost 8d. per square yard? *Ans.* 96 yards.

37. To solve ques. 31 and 36, Art. 60, by equations.

First. Let $x =$ the vol. at 32°; then the vol. at 42° $= x + \dfrac{x}{490} \times 10 = x + \dfrac{x}{49}$; but, from the question, this vol. is 100 c. in.;

$$\therefore\; x + \frac{x}{49} = 100; \text{ multiplying these equals by 49,}$$
$$49x + x = 4900; \; \therefore x = 98 \text{ c. in.}$$

Second. Let $x =$ the vol. at 29·5 pressure, and 32° temp.; then

$$\text{the vol. at 30·5 press., temp. } 32° = x \times \frac{29{\cdot}5}{30{\cdot}5} = \tfrac{59}{61} x;$$
$$\text{the vol. at 30·5 \;\;,, \;\;\;,, \;\; } 52° = \tfrac{59}{61} x + \tfrac{59}{61} x \times \tfrac{20}{490};$$

but, from the question, this volume is 118 c. in.;

$$\therefore\; \tfrac{59}{61} x + \tfrac{59}{61} x \times \tfrac{2}{49} = 118; \text{ multiplying by } 61 \times 49,$$
$$49 \times 59x + 59 \times 2x = 118 \times 61 \times 49; \; \therefore x = 117\tfrac{11}{51} \text{ c. in.}$$

38. Solve ques. 33, 37, 25, Art. 60, by equations.

39. A, B, C, travel from the same place at the rate of 3, 4, and 6 miles per hour respectively; if B starts 5 hours after A, how long

after B must C start, so that they may both overtake A at the same instant?

Let $x =$ the number hours B takes to overtake A; then
the distance gone by B, in miles $= 4\,x$
" " A, " $= 3\,(x + 5) = 3\,x + 15$;
but these distances are equal to each other;
$\therefore\ 4\,x = 3\,x + 15$; $\therefore\ x = 15$, number hours;
$\therefore$ the distance B must travel to overtake A $= 4 \times 15 = 60$ m.;
number hours for C to travel 60 miles $= 60 \div 6 = 10$;
but B took 15 hours to overtake A;
$\therefore$ number hours after B at which C must start $= 15 - 10 = 5$.
Verification. $3 \times (15 + 5) = 4 \times 15 = 6 \times (15 - 5)$.

40. If B starts 6 hours after A, how long after B must C start?
Ans. 6 hours.

41. A farmer bought some sheep for £36. After dividing them into three equal lots, he sold the first lot at a loss of 2s. a head, the second at a gain of 3s. a head, and the third at a gain of 4s. a head, and received £40 for the whole. How many sheep did he buy, and what price did he pay for each?

Gain per head $= \frac{1}{3}$ (3s. + 4s. − 2s.) $= \frac{5}{3}$s.
Gain on the whole = £40 − £36 = 80s.;
$\therefore$ number sheep = 80s. $\div \frac{5}{3}$s. $= 80 \times \frac{3}{5} = 48$;
and cost price each = £36 $\div$ 48 = 15s.

Or thus. Let $x =$ the number of sheep in each lot; then
the whole gain in shillings $= 3\,x + 4\,x - 2\,x = 5\,x$;
but the whole gain is 80s.; $\therefore\ 5\,x = 80$; $\therefore\ x = 16$;
$\therefore\ 3\,x = 48$, the total number of sheep; and so on.

42. Find the cost price of each sheep, supposing 1s. per head to be lost on the sale of the first lot. *Ans.* 18s.

43. Solve ques. 56, 57, Art. 40, by equations.

44. A person had some strong spirits to sell; if he mixed it with 3 gallons of water he could sell it at 18s. per gallon, and if he mixed it with 5 gallons of water he could sell it at 16s.; how many gallons of spirits were there?

Let $x =$ the number gallons; then
value of spirits $= 18\,(x + 3)$; also value spirits $= 16\,(x + 5)$;
$\therefore\ 18\,(x + 3) = 16\,(x + 5)$; performing the multiplication,
$18\,x + 54 = 16\,x + 80$; $\therefore\ x = 13$ gallons.

45. If the first mixture contained 2 gallons of water; how many gallons of spirits were there, and what would be the price per gallon?
Ans. 22, 19s. $7\frac{7}{11}$d.

46. To solve ques. 62, and 65, Art. 40, by equations.

First. Let $x =$ the number lbs. free luggage; then the number lbs. charged, in each case respectively, will be $392 - 2\,x$, $392 - x$; and *the charge per lb.*, determined for each case, will be $\frac{75}{392 - 2\,x}$ and $\frac{90}{392 - x}$; but these must be equal to each other, for they both express the value of the same thing;

$$\therefore \quad \frac{75}{392 - 2x} = \frac{90}{392 - x};$$

multiplying first by $392 - x$, and then by $392 - 2x$, we get

$75(392 - x) = 90(392 - 2x)$; dividing by 15,

$5(392 - x) = 6(392 - 2x)$; performing the multiplication,

$1960 - 5x = 2352 - 12x$; adding $12x$ to each, &c.,

$7x = 392$; $\therefore x = 56$ lbs.

Second. Let $x =$ the number lbs.; then

Cost price, in s. $= 5x - 15$; also, cost price, in s. $= 4\frac{1}{2}x - 7$;

$\therefore \; 5x - 15 = 4\frac{1}{2}x - 7$; adding 15 to these equals,

$5x = 4\frac{1}{2}x + 8$; multiplying these equals by 2,

$10x = 9x + 16$; $\therefore x = 16$.

47. Solve questions 63, 58, 59, 60, 61, 64, 66, 67, Art. 40, by equations.

48. A smuggler had a quantity of brandy which he expected would raise £12; after he had sold 8 gallons, a revenue officer seized $\frac{1}{3}$ of the remainder, in consequence of which he makes only £9 12s.; required the quantity of brandy and the price per gallon.

Let $x =$ the number gallons; then price per gallon, in s. $= \frac{240}{x}$; and number gallons seized $= \frac{1}{3}(x - 8)$;

$\therefore$ Value of the part seized, in s. $= \frac{1}{3}(x - 8) \times \frac{240}{x} = (x - 8)\frac{80}{x}$;

but the value of the part seized $=$ £12 $-$ £9 12s. $=$ 48s.

$\therefore (x - 8)\frac{80}{x} = 48$; mult. these equals by x,

$(x - 8)\,80 = 48x$; performing the mult.

$80x - 640 = 48x$; adding 640 to these equals, &c.,

$32x = 640$; $\therefore x = 20$ gals.;

and the price per gal. $=$ 240s. $\div$ 20 $=$ 12s.

49. Required the same as in the last example, supposing 4 gals. were sold and $\frac{1}{4}$ the remainder seized. *Ans.* 20 gals., 12s.

50. 6 ducks and 5 hens cost 28s., and 3 ducks and 4 hens cost 17s.; what is the price of a hen?

6 ducks + 5 hens = 28s. . . . (1.)

3 ducks + 4 hens = 17s. . . . (2.)

Let us double both sides of the second equation; then

6 ducks + 8 hens = 34s.;

but this equation gives the value of 3 hens more than eq. (1.);

$\therefore$ 3 hens = 34s. — 28s. = 6s.;

$\therefore$ Price 1 hen = $\frac{1}{3}$ of 6s. = 2s.

Or thus, by two unknown quantities. Let $x =$ the price of a duck, in shillings; and $y =$ the price of a hen in shillings; then

$6x + 5y = 28$ . . . (1.)

$3x + 4y = 17$ . . . (2.)

Doubling both sides of this latter equation, we get

$6x + 8y = 34$; taking eq. (1.) from this,

$3y = 6$; $\therefore y =$ 2s. price of a hen;

and putting this value of y in eq. (1.), we get

$6x + 5 \times 2 = 28$; $\therefore x = 18 \div 6 =$ 3s., price of a duck.

51. 12 bottles and 3 basins cost 31s., and 4 bottles and 9 basins cost 13s.; what is the price of each? *Ans.* 2s. 6d., and 4d.

52. A grocer bought tea at 3s. per lb., and coffee at 2s., and paid altogether 90s.; he sold the tea at 4s. and the coffee at 3s. and thereby gained 35s. How many lbs. of each did he buy?

Here the selling price = 90s. + 35s. = 125s. Putting x for the number of lbs. of tea, and y for the number of lbs. of coffee, we readily get the two following equations:

$$3x + 2y = 90 \ldots (1.)$$
$$4x + 3y = 125 \ldots (2.)$$

In order to obtain the same number of x's in both equations, we multiply the first by 4, and the second by 3.

Multiplying (1.) by 4, $12x + 8y = 360$
multiplying (2.) by 3, $12x + 9y = 375$
subtracting, $y = 15$, the no. lbs. of coffee;

and putting this value of y in equation (1.); we get

$$3x + 2 \times 15 = 90; \therefore x = 20, \text{ the no. lbs. of tea.}$$

53. If he bought the tea at 3s. 6d., and the coffee at 1s. 6d.; how many lbs. of each did he buy? *Ans.* $18\frac{1}{3}$, $17\frac{1}{2}$.

54. To solve ques. 44, Art. 69, and ques. 31, Art. 70, by two unknown quantities.

First. Putting x and y for the parts, we find

$$x + y = 10 \ldots (1.)$$
$$x - y = 4 \ldots (2.)$$

adding, $2x = 14$; $\therefore x = 7$.
Subtracting (2.) from (1.), $2y = 6$; $\therefore y = 3$.

Or thus. From (1.), $x = 10 - y$; from (2.), $x = 4 + y$;
$\therefore 4 + y = 10 - y$; $\therefore y = 3$; and so on.

Second. Putting x = the weight of the head, in lbs.; and y the weight of the body; we readily obtain the two following equations:

$$y = 4 + \frac{x}{2}; \text{ and } x = 4 + \frac{y}{5}.$$

By an easy reduction, these equations become

$$2y - x = 8 \ldots (1.); \ 5x - y = 20 \ldots (2.);$$

whence the values of x and y are obtained; and so on.

55. Solve by two unknown quantities, ques. 5, 45, 46, 47, 54, 55, Art. 69, and ques. 12, 13, 17, 32, 33, 34, Art. 70.

56. A square piece of board contains 81 square inches; find the side.

Let x = the length of the side, in inches; then the area in square inches $= x \times x = x^2$; but, by the question, the area is 81 square inches;

$\therefore x^2 = 81$; taking the square root of these equals,
$x = 9$ inches, the length of the side.

57. A box, 3 times as high as it is broad, and 5 times as long as it is high, contains 1215 cubic inches. Find the breadth of the box.

Let x = the breadth in inches; then the height $= 3x$; the length = 5 times the height = 5 times $3x = 15x$; and the content in cubic inches $= x \times 3x \times 15x = 45x^3$; but, by the question, the content is 1215 cubic inches;

$\therefore$ $45\,x^3 = 1215$; dividing these equals by 45,
$x^3 = 27$; taking the cube root of these equals,
$x = 3$ inches, the breadth required.

58. A rectangle, thrice as long as it is broad, contains 48 square feet; find the breadth. *Ans.* 4 feet.

59. The edges of a rectangular chest, which contains 216 cubic feet, are as 1, 2, and 4; find the length of the edges. *Ans.* 3, 6, 12 feet.

60. A certain number of soldiers could be formed into a solid square; if they were formed into a square, containing one man less in the side, there would be 13 men left over. Find the number of men.

Let $x =$ the number in the side of the less square; then $x + 1$ will be the number in the side of the greater; hence we have

The number men $= (x + 1)^2 = x^2 + 2\,x + 1$, see Art. 73.
Also, „ „ $= x^2 + 13$
$\therefore$ $x^2 + 2\,x + 1 = x^2 + 13$; taking x^2 from these equals,
$2\,x + 1 = 13$; taking 1 from these equals,
$2\,x = 12$; $\therefore x = 6$;
$\therefore$ *The number men* $= (x + 1)^2 = (6 + 1)^2 = 49$.

61. Supposing that there were 17 men left over, what would be the number? *Ans.* 81.

62. If 2 be added to a number its square will be 81.

Let $x =$ the number, then, by the question, we have
$(x + 2)^2 = 81$; taking the square root of these equals,
$x + 2 = 9$; $\therefore x = 7$.

63. If 3 be taken from a number its square will be 16. Find the number. *Ans.* 7.

64. The square of the half of a number exceeds the square of its fifth part by 21; what is the number?

Let $x =$ the number; its half $= \frac{x}{2}$; and its fifth part $= \frac{x}{5}$;

the square of $\frac{x}{2} = \frac{x}{2} \times \frac{x}{2} = \frac{x^2}{4}$; and the square of $\frac{x}{5} = \frac{x}{5} \times \frac{x}{5} = \frac{x^2}{25}$;

$\therefore$ $\frac{x^2}{4} - \frac{x^2}{25} = 21$; multiplying these equals by 100,
$25\,x^2 - 4\,x^2 = 2100$; performing the subtraction,
$21\,x^2 = 2100$; $\therefore x^2 = 100$; $\therefore x = 10$.

65. The square of a number added to the square of its third part is 40; find the number. *Ans.* 6.

66. The difference of two numbers is 8, and their product is 33; find the numbers.

Let $x =$ the less, then $x + 8 =$ the greater, and
their product $= (x + 8) \times x = x^2 + 8\,x$;
but, by the question, this is 33;
$\therefore$ $x^2 + 8\,x = 33$; adding 16 to these equals,*
$x^2 + 8\,x + 16 = 33 + 16 = 49$; taking the sq. root; Art. 73,
$x + 4 = 7$; $\therefore x = 3$, the less;
and the greater $= 3 + 8 = 11$.

* To make $x^2 + 8\,x$ an exact square, we add $(\frac{8}{2})^2$ or 4^2, that is, the square of half the coefficient of x. See Art. 73.

67. The difference of two numbers is 6, and their product is 16; find them. *Ans.* 8, 2.

68. The square of a number added to twice the number is 15; find the number. *Ans.* 3.

69. A person sold a horse for £11, and gained as much per cent. as the horse cost him. Find the cost price.

Let x = the cost price in £.; then, by the question,

Gain on £100 $= x$;

$$\therefore \quad \text{"} \quad x = \frac{x}{100} \times x = \frac{x^2}{100}; \ \therefore \text{ selling price} = \frac{x^2}{100} + x;$$

but, by the question, the selling price is £11,

$$\therefore \ \frac{x^2}{100} + x = 11; \text{ multiplying these equals by 100,}$$

$$x^2 + 100\,x = 1100; \text{ adding } 50^2 \text{ or } 2500,$$

$$x^2 + 100\,x + 2500 = 3600; \text{ taking the square root,}$$

$$x + 50 = 60; \ \therefore x = \text{£}10, \text{ cost price.}$$

70. What would be the cost price when the selling price is £24? *Ans.* £20.

71. The difference of two numbers is 3, and the difference of their squares is 39; find them.

Let x = the less, then $x + 3$ = the greater; the square of the less $= x \times x = x^2$; and the square of the greater $= (x + 3)^2 = x^2 + 6\,x + 9$. But, by the question, the difference of these must be equal to 39;

$$\therefore \ x^2 + 6\,x + 9 - x^2 = 39;$$

$$\therefore \ 6\,x + 9 = 39; \ \therefore x = 5, \text{ the less;}$$

and the greater $= 5 + 3 = 8$.

72. The difference of two numbers is 4, and the difference of their squares is 64; find them. *Ans.* 6, 10.

73. There are two numbers, one of which is double the other, and the difference of their squares is 27; find them. *Ans.* 3, 6.

74. A person bought a number of sheep for £24; he sold them for £28 and gained 4s. a head; how many were there? *Ans.* 20.

75. A and B set out at the same time for the same place, A travelling 2 miles an hour faster than B; A completes his journey in 6 hours and B in 9 hours; find the distance of the place. *Ans.* 36 miles.

INVOLUTION AND EVOLUTION, ETC.

71. By involution we find the *powers* of quantities, and by evolution we find their *roots*. When a quantity is multiplied by itself it is called the 2nd power or square of quantity; thus 3×3, or $3^2 = 9$, is called the 2nd power or square of 3. The product of three sevens, or $7 \times 7 \times 7$, or $7^3 = 343$, is called the 3rd power or cube of 7; and so on to other cases. The small figure, placed over the quantity, is called the *index* or *exponent*, and indicates the number of times the quantity is repeated; thus 3^5 means $3 \times 3 \times 3 \times 3 \times 3$. The square root of a quantity is that quantity whose square is equal to the given quantity; thus the square root of 25 is 5, because $5^2 = 25$. The cube root or 3rd

root of a quantity is that quantity whose cube or 3rd power is equal to the given quantity; thus the cube root or 3rd root of 8 is 2, because $2^3 = 8$; and so on to other roots. Roots are expressed by the symbol $\sqrt{\ }$; thus the 5th root of 32 is expressed by $\sqrt[5]{32} = 2$; the 3rd root or cube root of 125 is expressed by $\sqrt[3]{125} = 5$; the square root of 9 is expressed by $\sqrt[2]{9}$ or simply by $\sqrt{9} = 3$; and so on. Hence we have,—$\sqrt{5} \times \sqrt{5} = 5$, $\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = 8$; and so on. Thus roots and powers denote reverse operations, for example, the cube of the cube root of 8 is 8, or symbolically $(\sqrt[3]{8})^3 = 8$, or generally $(\sqrt[n]{a})^n = a$. A number whose root cannot be exactly found is called an *irrational quantity* or *surd;* thus $\sqrt{3}$ and $\sqrt[3]{7}$ are surds.

72. *Properties of Indices.*

1. $4^3 \times 4^2 = \overline{4 \times 4 \times 4} \times \overline{4 \times 4} = 4^5 = 1024$; *where we add the exponents of the factors to obtain the exponent of the product.* General formula,—$a^n \times a^m = a^{n+m}$. This formula holds true for fractional and negative indices, as well as for integer indices.

The symbol $\sqrt{a}$ may be written $a^{\frac{1}{2}}$, for $a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a$; and $\sqrt[3]{a}$ may be written $a^{\frac{1}{3}}$, for $a^{\frac{1}{3}} \times a^{\frac{1}{3}} \times a^{\frac{1}{3}} = a$, the addition of the exponents producing 1; and so on generally. When a fractional index is used with any quantity, the *numerator* denotes the *power* and the *denominator* the *root* to be taken; thus $a^{\frac{2}{3}}$ means $\sqrt[3]{a^2}$ or the cube root of the square of a.

2. *The power of a power* of a quantity is obtained by multiplying the indices of the two powers. Thus the cube of 4^2 or $(4^2)^3 = \overline{4 \times 4} \times \overline{4 \times 4} \times \overline{4 \times 4} = 4^6 = 4096$; and in like manner, $(3^{\frac{1}{2}})^6 = \overline{3^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times 3^{\frac{1}{2}}} = 3^3 = 27$. The general formula in this case is,—$(a^n)^m = a^{nm} = (a^m)^n$.

Hence we may shorten the operation of finding certain powers of quantities. Thus the 6th power of 3 may be found by squaring its cube, or $3^6 = (3^3)^2 = 27 \times 27 = 729$. The 9th power of 2 may be found by cubing the cube, or $2^9 = (2^3)^3 = 8^3 = 512$.

3. $(2 \times 4)^3 = \overline{2 \times 4} \times \overline{2 \times 4} \times \overline{2 \times 4} = 2^3 \times 4^3 = 512$. The general formula in this is,—$(a \times b)^n = a^n \times b^n$. Hence we may in some cases shorten the operation. Thus $4^3 \times 5^3 = (4 \times 5)^3 = 20^3 = 8000$.

4. $\dfrac{4^5}{4^3} = \dfrac{4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4} = 4 \times 4 = 4^2$; where we subtract the exponent of the divisor from the exponent of the dividend to obtain the exponent of the quotient. The general formula in this case is,—$\dfrac{a^n}{a^m} = a^{n-m}$.

73. To find an expression for the square of a number consisting of two terms. For example, let it be required to find the expression for the square of $6 + 4$.

Here 6 + 4 is taken 4 times and then 6 times, and the results added together. Showing that the square of 6 + 4 is equal to the square of the first term + twice the product of the two terms + the square of the second.

$$\begin{array}{rl} & 6 + 4 \\ & 6 + 4 \\ \hline & 6 \times 4 + 4^2 = (6 + 4) \times 4 \\ 6^2 + 6 \times 4 & \qquad\quad\; = (6 + 4) \times 6 \\ \hline 6^2 + 2 \times 6 \times 4 + 4^2 & = (6 + 4)^2 \end{array}$$

In like manner, $(a + b)^2 = a^2 + 2ab + b^2$. Also, $(a - b)^2 = a^2 - 2\,ab + b^2$.

Conversely the square root of $6^2 + 2 \times 6 \times 4 + 4^2$ is $6 + 4$; and generally the square root of $a^2 + 2\,ab + b^2$ is $a + b$.

The quantity $x^2 + 2\,ax$ may be made an exact square by adding a^2 to it, that is, by adding the square of half the coefficient of x. Thus $x^2 + 6\,x$ may be made an exact square by adding 3^2 or 9, and then the square root of $x^2 + 6\,x + 9$ will be $x + 3$. This operation is useful in solving quadratic equations.

By multiplication, we also have, $(a + b)^3 = a^3 + 3\,a^2b + 3\,ab^2 + b^3$; and so on to other powers.

74. *Extraction of the second, or square root.*

The square root of the following quantities is at once found: $\sqrt{81} = 9$; $\sqrt{49} = 7$; $\sqrt{\frac{1}{4}} = \frac{1}{2}$; $\sqrt{\frac{4}{9}} = \frac{2}{3}$; $\sqrt{\frac{49}{64}} = \frac{7}{8}$. But when the square root of a quantity contains two or more figures the operation is conducted as follows:

To find the square root of 745·29 and ·09365.

$\sqrt{\dot{7}4\dot{5}2\dot{9}} = 27{\cdot}3$	
	4
47)	345
	329
543)	1629
	1629
	....

$\sqrt{\dot{0}{\cdot}0\dot{9}3\dot{6}5\dot{0}} = {\cdot}306023$, &c.	
	9
606)	3650
	3636
60202)	140000
	120404
602043)	1959600
	&c., &c.

Here we first place a point over *the units' place*, then over every alternate figure. We take the nearest square root of the 7 (the figures within the first point) which will be 2 with 3 as a remainder. We now bring down 45, the figures included by the next point; and double the 2, the root figure found, which will make 4, this is the *trial divisor;* then 34 divided by 4 gives 8, but after completing the true divisor this root figure would be too great, therefore we put down 7 for the next figure in the root; we put this 7 also after the 4, and it gives us 47 for the *true divisor;* we then multiply 47 by 7 and subtract the product from the preceding remainder, leaving 16; we now bring down 29, and multiply 27, the last figures in the root, by 2, making 54 (or what is the same thing we add the 7 to 47) this gives us the new *trial divisor;* then 162 divided by 54 gives 3 for the next figure in the root; we put this 3 also

after the 54, which gives 543 for the *true divisor*; and so on. It will be observed that the *trial divisor* is always found by doubling the figures found in the root, and the new root figure annexed to the trial divisor gives the true divisor. In the second example there is no exact root, but we approximate to it as far as we please by taking down cyphers as they are wanted. Also, in this example, we have an instance of cyphers occurring in the root.

Proof of the rule. The first example is here written out in full.

$$\begin{array}{rl}
 & \sqrt{\dot{7}4\dot{5}2\dot{9}} = 200 + 70 + 3 \\
 & \;\;40000 = 200^2 \\
 & \overline{} \\
2 \times 200 + 70 = 470) & 34529 \\
 & 32900 = (2 \times 200 + 70)\,70 \\
 & \overline{} \\
2 \times 270 + 3 = 543) & \;\;1629 \\
 & \;\;1629 = (2 \times 270 + 3)\,3 \\
 & \overline{} \\
 & \;\;\cdots\cdot
\end{array}$$

It is obvious, that a quantity having *one* or *two* figures has *one* figure in the root, and that a quantity of *three* or *four* figures has *two* figures in the root, and so on; hence the number of dots always indicates the number of figures in the root. Here the number of dots is three, therefore there are three figures in the root. The rule for the extraction of the square root is derived from the *form* of the square of a binomial; thus we have by multiplication,

$$(a + z)^2 = a^2 + 2\,az + z^2 = a^2 + (2\,a + z)z;$$

making $a = 200$, $z = 70$; and $a = 270$, $z = 3$; we get

$$(200 + 70)^2 = 200^2 + (2 \times 200 + 70)\,70; \text{ and}$$
$$(270 + 3)^2 = 270^2 + (2 \times 270 + 3)3.$$

The nearest square root of 7 is 2, and the root of the proposed number will lie between 200 and 300, therefore we take 200 for the first term of the root. Let z be the next term of the root; then

$$(200 + z)^2 = 200^2 + 2 \times 200 \times z + z^2 = 200^2 + (2 \times 200 + z)z;$$

where it appears that, after subtracting 200^2 from the given number, we obtain z by dividing by the *trial divisor* 2×200, and then for the *true divisor* we add z the root thus found. In this example we find $z = 70$; then after subtracting the product of the true divisor by 70, we shall have subtracted, in all, $200^2 + (2 \times 200 + 70)\,70$, or $(200 + 70)^2$, or 270^2. We now find 2×270 for the next *trial divisor*, and the next term of the root to be 3, and the *true divisor* to be $2 \times 270 + 3$, which being multiplied by 3 and subtracted, leaves no remainder; but we shall have subtracted, in all, $270^2 + (2 \times 270 + 3)3$, or $(270 + 3)^2$ or 273^2. In precisely the same manner the proof may be extended to 4 figures in the root, and so on. In practice the redundant cyphers here employed are omitted, and instead of bringing down the whole remainder at every step, each period is taken down as it is wanted.

The square root of a fraction is equal to the square root of the numerator divided by the square root of the denominator; thus $\sqrt{\frac{9}{25}} = \frac{3}{5}$. But when these roots cannot be exactly found, the fraction should be brought to a decimal, before the square root is taken; thus $\sqrt{\frac{3}{8}} = \sqrt{\cdot 375}$

$= \cdot 6123$, &c. The roots of fractions may also be found as follows:— $\sqrt{\frac{2}{7}} = \sqrt{\frac{2 \times 7}{7 \times 7}} = \frac{1}{7}\sqrt{14} = \frac{1}{7}$ of $3 \cdot 7416$, &c. $= \cdot 5345$, &c.; again, $\sqrt{\frac{3}{8}} = \sqrt{\frac{3 \times 2}{8 \times 2}} = \frac{1}{4}\sqrt{6} = \frac{1}{4}$ of $2 \cdot 4494$, &c. $= \cdot 6123$, &c.

The square root of a decimal quantity can always be brought to the square root of an integer, as follows: $\sqrt{\cdot 003} = \sqrt{\frac{30}{10000}} = \frac{1}{100}\sqrt{30} = \frac{1}{100}$ of $5 \cdot 47$, &c. $= \cdot 0547$, &c.; again, $\sqrt{\cdot 02} = \sqrt{\frac{2}{100}} = \frac{1}{10}\sqrt{2} = \cdot 1414$, &c.

Examples. Find the square root of 132496; 25·4016; 5·12; ·34; 7; $\frac{2}{3}$; $\frac{1}{5}$; $\frac{3}{4}$; ·0002.

Answers. 364; 5·04; 2·2627, &c.; ·583, &c.; 2·6457, &c.; ·8164, &c.; ·449, &c.; ·866, &c.; ·01414, &c.

75. *Extraction of the third, or cube root.*

The cube root of the following quantities is at once found: $\sqrt[3]{125} = 5$; $\sqrt[3]{343} = 7$; and the cube root of the fraction $\frac{64}{125} = \frac{4}{5}$. But when the cube root of a quantity contains two or more figures the operation is conducted as follows:—

To find the cube root of 83·6238.

$$
\begin{array}{rr}
3 \times 4^2 = & 48 \\
3 \times 4 \times 3 = & 36 \\
3^2 = & 9 \\
\hline
 & 5169 \\
 & 369 \\
\hline
3 \times 43^2 = & 5547 \\
3 \times 43 \times 7 = & 903 \\
7^2 = & 49 \\
\hline
 & 563779 \\
 & 9079 \\
\hline
3 \times 437^2 = & 572907 \\
3 \times 437 \times 2 = & 2622 \\
2^2 = & 4 \\
\hline
 & 57316924
\end{array}
$$

$$
\begin{array}{l}
\sqrt[3]{8\dot{3} \cdot 62\dot{3}80\dot{0}} = 4 \cdot 372, \text{ \&c.} \\
64 \\
\hline
19623 \\
15507 \\
\hline
4116800 \\
3946453 \\
\hline
170347000 \\
114633848 \\
\hline
55713152000 \\
\qquad\qquad \text{\&c., \&c.}
\end{array}
$$

Here we first place a point over the units' place, and then over every third figure. We take the nearest cube root of 83, which will be 4 with 19 as a remainder. We bring down 623, the figures included by the next point; and take 3 times the square of the root figure making 48; this is the *trial divisor*, then 196 divided by 48 gives 4, but after completing the true divisor this root figure would be too great, therefore we put down 3 for the next figure in the root. To complete the divisor we take 3 times the preceding root figure multiplied by the new figure of the root, and write the product, 36, beneath the 48, taking care to remove the figures a place to the right; in the same manner we write down the square of the new root figure; these added together give 5169

for the *true divisor*, which multiplied by 3, and subtracted from the subtrahend 19623, gives 4116 as a remainder; and so on to all the other figures of the root. It remains to be shown how the next trial divisor is found without the trouble of squaring 43 and multiplying by 3. Bring down 36 and 9 making 369, and then add the three lines of figures connected by the bracket, and this will give 5547 for the trial divisor; and so on to all the succeeding ones. In this example there is no exact root, but we approximate to it as far as we please by taking down three cyphers as they are wanted. Two more figures (at least) in the root may be obtained by simply dividing the last subtrahend by the last divisor; thus we find the root to be 4·37297, &c.

Proof of the rule. To find the cube root of 94818816 retaining all the cyphers.

$$\begin{array}{rr}
3 \times 400^2 = & 480000 \\
3 \times 400 \times 50 = & 60000 \\
50^2 = & 2500 \\
\hline
 & 542500 \\
 & 62500 \\
\hline
3 \times 450^2 = & 607500 \\
3 \times 450 \times 6^2 = & 8100 \\
6^2 = & 36 \\
\hline
 & 615636
\end{array}
\qquad
\begin{array}{rl}
\sqrt[3]{9\dot{4}818\dot{8}1\dot{6} } & = 400 + 50 + 6 \\
64000000 & = 400^3 \\
\hline
30818816 & \\
27125000 & \\
\hline
3693816 & \\
3693816 & \\
\hline
\cdots\cdots\cdot &
\end{array}$$

It is obvious, that a quantity having *one* or *three* figures has *one* figure in the root, and that a quantity of *four* or *six* figures has *two* figures in the root, and so on; hence the number of dots always indicates the number of figures in the root. Here the number of dots is three, therefore there are three figures in the root. The rule for the extraction of the cube root is derived from the form of the cube of a binomial; thus we have by multiplication,

$$(a + z)^3 = a^3 + 3\,a^2z + 3\,az^2 + z^3 = a^3 + (3\,a^2 + 3\,az + z^2)z;$$

making $a = 400$, $z = 50$; and $a = 450$, $z = 6$; we get

$$450^3 = (400 + 50)^3 = 400^3 + (3 \times 400^2 + 3 \times 400 \times 50 + 50^2)\,50;$$
$$456^3 = (450 + 6)^3 = 450^3 + (3 \times 450^2 + 3 \times 450 \times 6 + 6^2)\,6.$$

Having found 400 for the first term of the root, let z be the next term of the root, then

$$(400 + z)^3 = 400^3 + (3 \times 400^2 + 3 \times 400 \times z + z^2)z.$$

Where it appears that, after subtracting 400^3 from the given number, we have for finding z, the next term of the root, the *trial divisor* 3×400^2; then to complete the *true divisor*, to this trial divisor we add $3 \times 400 \times$ the last term of the root + the square of the last term of the root. In this example we find $z = 50$; then after subtracting the product of the true divisor by 50, we shall have subtracted, in all, $400^3 + (3 \times 400^2 + 3 \times 400 \times 50 + 50^2)\,50$, or $(400 + 50)^3$, or 450^3. We now find 3×450^2 for the next *trial divisor* (the manner of obtaining this will be afterwards explained) and the next term of the root to be 6, and the true divisor to be $3 \times 450^2 + 3 \times 450 \times 6 + 6^2$, or 615636, which multiplied by 6 and subtracted leaves no remainder, but we shall

have now subtracted, in all, $450^3 + (3 \times 450^2 + 3 \times 450 \times 6 + 6^2)\, 6$, or 456^3. It remains for us to show how the trial divisor is found by the easy process given in the rule. For the second trial divisor, for example, we have

$$\begin{aligned} 3 \times 450^2 &= 3\,(400 + 50)^2 = 3 \times 400^2 + 3 \times 2 \times 400 \times 50 + 3 \times 50^2 \\ &= 3 \times 400^2 + 3 \times 400 \times 50 + 50^2 + 3 \times 400 \times 50 + 2 \times 50^2. \end{aligned}$$

This formula is, in fact, the operation given in the rule, where the first three terms are the preceding trial divisor, and the last two terms are the addenda forming the new trial divisor. The same process will obviously apply to any other trial divisor.

76. The cube root of a fraction is equal to the cube root of the numerator divided by the cube root of the denominator; thus the cube root of $\frac{27}{64} = \frac{3}{4}$. But when these roots cannot be exactly found, the fraction should be brought to a decimal, before the root is taken; thus $\sqrt[3]{\frac{3}{5}} = \sqrt[3]{\cdot 6} = \cdot 8434$, &c. The cube root of a fraction may also be found as follows: $\sqrt[3]{\frac{2}{3}} = \sqrt[3]{\frac{2 \times 3 \times 3}{3 \times 3 \times 3}} = \frac{1}{3}\sqrt[3]{18} = \frac{1}{3}$ of $2\cdot 6207$, &c. $= \cdot 8735$, &c. The cube root of a decimal can always be brought to the cube root of an integer, as follows: $\sqrt[3]{\cdot 21} = \sqrt[3]{\frac{210}{1000}} = \frac{1}{10}\sqrt[3]{210} = \frac{1}{10}$ of $5\cdot 9439$, &c. $= \cdot 59439$, &c.

Examples. Find the cube root of 440711081; 344·472101; 645420545288; 5; $\frac{1}{3}$; 28; 3; $\frac{1}{5}$.

Answers. 761; 7·01; 8642; 1·7099, &c.; ·69336, &c.; 3·0365, &c.; 1·4422, &c.; ·5848, &c.

77. *Properties of roots.*

1. General formula:—$(a^{\frac{1}{n}})^{\frac{1}{m}} = (a^{\frac{1}{m}})^{\frac{1}{n}} = a^{\frac{1}{mn}}$.

$$a = (a^{\frac{1}{6}})^6;\ \therefore a^{\frac{1}{2}} = (a^{\frac{1}{6}})^3;\ \therefore (a^{\frac{1}{2}})^{\frac{1}{3}} = a^{\frac{1}{6}};$$

that is, the sixth root of a quantity may be found by taking the cube root of the square root of the quantity, or by reversing these operations; thus $\sqrt[6]{64} = \sqrt[3]{8} = 2$, or $\sqrt[6]{64} = \sqrt{4} = 2$; $\sqrt[6]{15625} = \sqrt{25} = 5$.

Similarly $(a^{\frac{1}{2}})^{\frac{1}{2}} = a^{\frac{1}{4}}$; that is, the fourth root may be found by taking the square root of the square root; thus $\sqrt[4]{1296} = \sqrt{36} = 6$.

Similarly $(a^{\frac{1}{3}})^{\frac{1}{3}} = a^{\frac{1}{9}}$; that is, the ninth root may be found by taking the cube root of the cube root; thus $\sqrt[9]{12812904} = \sqrt[3]{234} = 6\cdot 162$, &c. And so on to other roots.

2. General formula:—$a^{\frac{1}{n}} \times a^{\frac{1}{m}} = a^{\frac{1}{n} + \frac{1}{m}} = a^{\frac{m+n}{mn}}$.

$(a^{\frac{1}{2}} \times a^{\frac{1}{3}})^6 = (a^{\frac{1}{2}})^6 \times (a^{\frac{1}{3}})^6 = a^3 \times a^2 = a^5$; taking the sixth root,

$$a^{\frac{1}{2}} \times a^{\frac{1}{3}} = a^{\frac{5}{6}} = (a^5)^{\frac{1}{6}} = (a^{\frac{1}{6}})^5.$$

Thus $\sqrt{4096} \times \sqrt[3]{4096} = 4096^{\frac{5}{6}} = 4^5 = 1024$.

3. General formula:—$a^{\frac{1}{n}} \times b^{\frac{1}{n}} = (a \times b)^{\frac{1}{n}}$.

$(a^{\frac{1}{3}} \times b^{\frac{1}{3}})^3 = (a^{\frac{1}{3}})^3 \times (b^{\frac{1}{3}})^3 = a \times b$; taking the third root, $a^{\frac{1}{3}} \times b^{\frac{1}{3}} = (a \times b)^{\frac{1}{3}}$. Thus $\sqrt[3]{56} = \sqrt[3]{8 \times 7} = \sqrt[3]{8} \times \sqrt[3]{7} = 2\sqrt[3]{7}$. Similarly $\sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$; and $\sqrt{12} + \sqrt{48} = \sqrt{4} \times \sqrt{3} + \sqrt{16} \times \sqrt{3} = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$.

Exercises. $\sqrt[4]{456976} = 26$; $\sqrt[6]{594823321} = 29$; $\sqrt[9]{40353607} = 7$; $\sqrt[4]{54289} = 15{\cdot}264$, &c.; $\sqrt[9]{141420761} = 8{\cdot}046$, &c.; $\sqrt{64} \times \sqrt[3]{64} =$: $= 32$; $\sqrt{108} - \sqrt{12} = 4\sqrt{3}$; $\sqrt[3]{54} + \sqrt[3]{16} = 5\sqrt[3]{2}$.

78. *Calculation by Logarithms.*

Arithmetical calculations are much simplified by the use of logarithm The multiplication of numbers is found by adding their logarithms, an the division of one number by another is found by taking the differenc of their logarithms. Any power of a number is found by multiplyin the logarithm of that number by the given power, and any root of number is found by dividing the logarithm of that number by the figur of the root. The manner of finding the logarithm of a given numbe and reversely of finding the number corresponding to a given logarithn is sufficiently explained in all books containing tables of logarithms.

To multiply 5·3472 by 12·613.

Here we add the logarithms of the numbers, and then find, in the tables, the number corresponding to the logarithm, 1·828, &c., to be 67·444, &c., which is the product required.

$$\begin{array}{rl} \log. 5{\cdot}3472 = & {\cdot}7281264 \\ \log. 12{\cdot}613 = & 1{\cdot}1008184 \\ \hline & 1{\cdot}8289448 \\ & = \log. 67{\cdot}444, \&c. \end{array}$$

To divide 246·45 by 12·301.

Here we subtract the logarithm of the divisor from the logarithm of the dividend, and then find, in the tables, the number corresponding to the logarithm, 1·3017, &c., to be 20·034, &c., which is the quotient required.

$$\begin{array}{rl} \log. 246{\cdot}45 = & 2{\cdot}3917288 \\ \log. 12{\cdot}301 = & 1{\cdot}0899404 \\ \hline & 1{\cdot}3017885 \\ & = \log. 20{\cdot}034, \&c. \end{array}$$

To find the 20th power of 1·0232.

Here we multiply the logarithm of the number by 20, and then find, in the tables, the number corresponding to the logarithm, ·1992, &c., to be 1·582, &c., which is the power required.

$$\begin{array}{rl} \log. 1{\cdot}0232 = & {\cdot}0099605 \\ & 20 \\ \hline & {\cdot}1992100 \\ & = \log. 1{\cdot}582, \&c. \end{array}$$

To find the 3rd root of 83·623.

Here we divide the logarithm of the number by 3, and then find, in the tables, the number corresponding to the logarithm, ·64077, &c., to be 4·3729, &c., which is the cube root required.

$$\begin{array}{rl} \log. 83{\cdot}623 = & 1{\cdot}9223257 \\ \text{Its quotient by } 3 = & {\cdot}6407752 \\ & = \log. 4{\cdot}3729, \&c. \end{array}$$

To find the value of $£\dfrac{12{\cdot}34 \times 1{\cdot}04^5}{6{\cdot}213}$.

Here we add together the logarithms of the two quantities in the numerator, and subtract the logarithm of the denominator.

$$\begin{aligned} \log.\ 12{\cdot}34 &= 1{\cdot}0913152 \\ \log.\ 1{\cdot}04^5 = 5 \times {\cdot}0170333 &= \ \ {\cdot}0851665 \\ \hline &\ \ \ 1{\cdot}1764817 \\ \log.\ 6{\cdot}213 &= \ \ {\cdot}7933014 \\ \hline &\ \ \ {\cdot}3831803 = \log.\ 2{\cdot}4164. \end{aligned}$$

Hence the value of the quantity is £2·4164 = £2 8s. $3\frac{3}{4}$d.

ARITHMETICAL PROGRESSION.

79. An arithmetical series is a collection of quantities, constantly increasing, or decreasing, from one to the other by a certain number called the common difference. Thus, $2 + 5 + 8 + 11 + 14$, is an arithmetical series, where the number of terms is 5, the first term 2, the fifth or last term 14, and the common difference 3. As any term is 3 greater than the term going before it, we may write the terms in the following form:—

1st term $= 2$; 2nd term $= 2 + 3$; 3rd term $= 2 + 2$ times 3; &c.;

where the number of 3's in any term is one less than the number indicating the place of that term; hence any proposed term may be readily written down, without the trouble of calculating the terms going before it; thus the 9th term $= 2 + (9 - 1)\,3 = 2 + 8 \times 3 = 26$.

1. Find the 11th term of the series $1 + 6 + 11 +$ &c. *Ans.* 51.

2. The first term of an arithmetical series is 7, and the common difference 8; required the 5th and the 13th terms. *Ans.* 39 and 103.

The sum of a series, s, is the addition of all the terms composing it. Let it be required to find the sum of the following arithmetical series:

$$s = 1 + 4 + 7 + 10 + 13 + 16.$$

Here writing the terms of the series in an inverse order,

$$s = 16 + 13 + 10 + 7 + 4 + 1,$$

Adding these two series together, we find

$$2\,s = 17 + 17 + 17 + 17 + 17 + 17 = 17 \times 6;$$

$$\therefore\ s = 17 \times \tfrac{6}{2} = (1 + 16)\,\tfrac{6}{2} = 51.$$

This result shows that *the sum of the series is equal to the sum of the first and last terms multiplied by half the number of terms.*

3. Find the sum of the series $3 + 7 + 11 +$ &c., to 20 terms.

Here the first term is 3; the common difference 4; the number of terms 20; and the 20th or last term $= 3 + 19 \times 4 = 79$; hence

the sum of the series $= (3 + 79)\,\frac{20}{2} = 82 \times 10 = 820$.

4. Find the sum of $2 + 5 + 8 +$ &c., to 16 terms. *Ans.* 392.

5. Find the sum of $5 + 6 + 7 +$ &c., to 19 terms. *Ans.* 266.

6. How many strokes does a clock strike in 7 days?

No. strokes in 12 hours $= 1 + 2 + \ldots + 12 = 13 \times \frac{12}{2} = 78$

$\therefore$ " 7 days $= 78 \times 2 \times 7 = 1092$.

7. A gardener planted some trees in the form of an isosceles triangle, putting 1 in the first row, 3 in the second, 5 in the third, and so on for 20 rows; required the number of trees. *Ans.* 400.

8. If 100 stones be laid in a line, at the distance of 6 feet from each other; how far must a person travel who picks them up singly and places them in a heap at the distance of 10 feet from the first stone?

Here, taking the distance one way, the first term is 10, the common difference 6, the number of terms 100; and the 100th or last term $= 10 + 99 \times 6 = 604$;

$\therefore$ the sum of the series $= (10 + 604) \frac{100}{2} = 30700$; and the total distance travelled $= 30700 \times 2 = 61400$ feet.

9. How far would he travel supposing the stones 5 feet apart? *Ans.* 50500 feet.

10. The first term of a arithmetical series is 8, the last term 53, and the number of terms 10; required the common difference.

The last term $= 8 + 9 \times$ com. diff.; but the last term is 53,

$\therefore$ $8 + 9 \times$ com. diff. $= 53$; $\therefore$ com. diff. $= \frac{1}{9}(53 - 8) = 5$.

11. Supposing the last term 62; what will be the common difference? *Ans.* 6.

12. The first term of an arithmetical progression is 2, the sum of the series 57, and the number of terms 6; required the last term.

Here, the sum of the series $= (2 + \text{last term}) \frac{6}{2}$;

$\therefore$ $(2 + \text{last term}) \frac{6}{2} = 57$; $\therefore$ last term $= 17$.

13. If the last term be 13, the number of terms 6, and the sum of the series 48; what will be the first term? *Ans.* 3.

14. The 1st and 5th terms of an arithmetical progression are 6 and 14 respectively; required the intermediate terms.

The 5th term $= 6 + 4 \times$ com. diff.; but the 5th term is 14,

$\therefore$ $6 + 4 \times$ com. diff. $= 14$; $\therefore$ com. diff. $= 2$;

and therefore the intermediate terms are 8, 10, 12.

15. Required the intermediate terms, when the 1st and 6th terms are 1 and 26 respectively. *Ans.* 6, 11, 16, 21.

GEOMETRICAL PROGRESSION.

80. In a geometrical series, any one of the terms is obtained from its preceding term by multiplying by a certain number called the *common ratio.* Thus, $2 + 6 + 18 + 54 +$ &c., is a geometrical series, where the 2nd term is obtained from the 1st, by multiplying by 3; the 3rd term from the 2nd by multiplying by 3; and so on. In this case the common ratio is 3. Having given the first term, the common ratio, and the number of terms, the series may be written down. Thus, if the first term be 3, the common ratio 2, and the number of terms 4, we shall have the series, $3 + 3 \cdot 2 + 3 \cdot 2^2 + 3 \cdot 2^3$, where the exponent of the 2, in any term, is one less than its place in the series; thus, in the 3rd term, this exponent is 2; in the 4th term it is 3; and so on. Hence any proposed term of this series may be readily found; thus, the 9th term $= 3 \times 2^8 = 768$.

1. Required the 5th term of the series $2 + 6 + 18 +$ &c. *Ans.* 162.

2. Required the 7th term of $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} +$ &c. *Ans.* $\frac{1}{128}$.

To find the sum of a geometrical series. Let it be required to find **the sum**, s, of six terms of the following series,

$$s = 3 + 12 + 48 + 192 + 768;\ \text{mult. by the ratio,}$$
$$4\,s = 12 + 48 + 192 + 768 + 768 \times 4;\ \text{subtracting,}$$
$$3\,s = 768 \times 4 - 3;\ \therefore s = \tfrac{1}{3}(768 \times 4 - 3) = 1023.$$

From this result we derive the following rule for finding the sum of a geometrical series.

Rule. Multiply the last term by the ratio, and divide the difference between this product and the first term, by the difference of the ratio and unity. General formula, $a + a\,r + a\,r^2 + \ldots + a\,r^{n-1} = \dfrac{a\,r^n - a}{r - 1}$.

When the ratio is a fraction and the number of terms is *infinite*, the last term is to be regarded as nothing, then the foregoing rule simply becomes,—*Divide the first term by the difference of the ratio and unity.* For example, let it be required to find the sum of the following series taken to infinity;

$$s = 3 + 1 + \tfrac{1}{3} + \tfrac{1}{9} + \text{\&c. to infinity; mult. by the ratio } \tfrac{1}{3},$$
$$\tfrac{1}{3}\,s = 1 + \tfrac{1}{3} + \tfrac{1}{9} + \tfrac{1}{27} + \text{\&c. to infinity; subtracting}$$
$$\tfrac{2}{3}\,s = 3;\ \therefore s = 3 \div \tfrac{2}{3} = 4\tfrac{1}{2};$$

where 3 is the first term, $\frac{1}{3}$ is the ratio, and $\frac{2}{3}$ is equal to $1 - \frac{1}{3}$; hence the rule.

3. Find the sum of the series $1 + 3 + 9 +$ &c., to 7 terms.

Here the 1st term $= 1$, the ratio $= 3$, the 7th or last term $= 1 \times 3^6 = 729$, and the difference of the ratio and unity $= 3 - 1 = 2$; hence we have, by the rule, the sum $= \frac{1}{2}(729 \times 3 - 1) = 1093$. But the student should also solve these questions from the first principles.

4. Find the sum of the series $\frac{1}{2} + \frac{1}{6} + \frac{1}{18} +$ &c. to 5 terms, also find the sum when the terms are taken to infinity.

Here the 5th term $= \frac{1}{2} \times (\frac{1}{3})^4 = \frac{1}{162}$; hence by the rule,

$$s = (\tfrac{1}{2} - \tfrac{1}{162} \times \tfrac{1}{3}) \div (1 - \tfrac{1}{3}) = \tfrac{242}{486} \times \tfrac{3}{2} = \tfrac{121}{162}.$$

By the rule for the infinite series, $s = \frac{1}{2} \div (1 - \frac{1}{3}) = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$.

5. Find the sum of the series, $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} +$ &c., first to 5 terms and then to infinity. *Ans.* $\frac{31}{32}$, and 1.

6. Find the sum of the series, $2{\cdot}5 + {\cdot}25 + {\cdot}025 +$ &c., first to 5 terms and then to infinity. *Ans.* $2{\cdot}77775$, and $2{\cdot}\dot{7}$.

7. Find the sum of $\frac{1}{2} + 2 + 8 +$ &c. to 7 terms. *Ans.* $2730{\cdot}5$.

8. Find the sum of $1{\cdot}05 + 1{\cdot}05^2 + 1{\cdot}05^3 + 1{\cdot}05^4$.

Here the ratio is $1{\cdot}05$, and $1{\cdot}05 - 1 = {\cdot}05$, therefore by the rule,

$$s = (1{\cdot}05^5 - 1{\cdot}05) \div {\cdot}05 = 4{\cdot}5256, \text{\&c.}$$

9. Find the sum of $2 + 2^2 + 2^3 +$ &c. to 12 terms. *Ans.* 8190.

10. Find the value of the recurring decimal ${\cdot}\dot{2}\dot{7}$.

$$s = {\cdot}2727' \text{ to infinity.}$$

Here, as there are two recurring figures, the common ratio will be $\frac{1}{100}$, and multiplying both sides of the equality by 100, we get

$$100\,s = 27{\cdot}27 \text{ to infinity; subtracting}$$
$$99\,s = 27;\ \therefore s = \tfrac{27}{99} = \tfrac{3}{11}.$$

11. Find the values of ${\cdot}\dot{5}$; ${\cdot}\dot{3}\dot{6}$; ${\cdot}\dot{1}3\dot{5}$. *Ans.* $\frac{5}{9}$, $\frac{4}{11}$, $\frac{5}{37}$.

12. What would be the price of a horse, which is sold at 1d. for the first nail in the shoes, 2d. for the second nail, 4d. for the third nail, and so on, allowing 24 nails in all?

Here the common ratio is 2 and the number of terms 24; therefore, the sum of the following geometrical series will give the price of the horse in pence.

$$s = 1 + 2 + 4 + \ldots + 2^{22} + 2^{23};$$
$$\therefore\quad 2s = 2 + 4 + 8 + \ldots + 2^{23} + 2^{24};$$
$$\therefore\quad s = 2^{24} - 1 = 16777215\text{d.} = £69905\ 1\text{s. } 3\text{d.}$$

13. What would be the price of the horse, allowing 12 nails in all?
Ans. £17 1s. 3d.

14. The 1st and 5th terms of a geometrical progression are 3 and 48 respectively; find the three intermediate terms.

Here, the 5th term $= 3 \times \text{ratio}^4$; but the 5th term is 48;

$$\therefore\quad 3 \times \text{ratio}^4 = 48;\ \therefore \text{ratio} = \sqrt[4]{16} = 2;$$

and the intermediate terms are 3×2, 3×4, 3×8, or 6, 12, 24.

15. Required the intermediate terms, when the 1st and fourth terms are 5 and 135 respectively. *Ans.* 15 and 45.

COMPOUND INTEREST.

81. When a person receives compound interest upon a sum of money, the interest, as it becomes due, is put to the principal, and then the succeeding interest is chargeable on this sum. Thus if the interest of £100 for 1 year be £5, then if this interest be put to the £100, there will be a principal of £105, upon which interest will be chargeable for the next year, and so on.

For example, let it be required to find the amount of £8 at compound interest for 3 years, at the rate of 5 per cent. per annum.

As 5 is the $\frac{1}{20}$ of 100, the interest of any principal for a year will be $\frac{1}{20}$ of that principal, and therefore the amount at the end of a year will be the principal added to $\frac{1}{20}$ of the principal, or $1\frac{1}{20}$ times the principal. It must be further observed, that the amount of £1 for a year will be £$1\frac{1}{20}$. Hence we have the following solution.

	£8·0	Principal.
For $\frac{1}{20}$	·4	Int. of the 1st year.
1st year's amount =	8·4	$= 8 \times 1\frac{1}{20}$.
For $\frac{1}{20}$	·42	Int. of the 2nd year.
2nd year's amount =	8·82	$= 8 \times 1\frac{1}{20} \times 1\frac{1}{20}$.
For $\frac{1}{20}$	·441	Int. of the third year.
3rd year's amount =	9·261	$= 8 \times 1\frac{1}{20} \times 1\frac{1}{20} \times 1\frac{1}{20}$.
=	£9 5s. $2\frac{16}{25}$d.	

The whole interest = £9 5s. $2\frac{16}{25}$d. — £8 = £1 5s. $2\frac{16}{25}$d.

Now $1\frac{1}{20}$ is the amount of £1 for a year; hence it follows, that *the* 3rd

year's amount = the principal × the 3rd power of the amount of £1 for a year; and so on for any other number of years.

If the interest is payable half-yearly the amount of £1 for a half-year will be £$1\frac{1}{40}$, then proceeding as before, we find, *the 3rd half-year's amount* $= 8 \times 1\frac{1}{40} \times 1\frac{1}{40} \times 1\frac{1}{40}$; that is, the 3rd half-year's amount = *the principal × the 3rd power of the amount of £1 for a half-year.*

Hence we have the following general rule for the calculation of compound interest, the interest being payable at any specified term.

Rule I.—Find the amount of £1 for the term at which interest is payable, involve this to a power denoted by the number of terms at which interest is payable; multiply this power by the principal and the product will be the amount. The principal subtracted from the amount will give the interest. The general formula is, $M = P(1 + R)^n$, where P is the principal, $1 + R$ the amount of £1 for the term at which interest is payable, and n the number of terms.

1. Find the amount of £50 for 5 years, at 4 per cent. per annum.

Here, the amount of £1 for a year $= 1 + \frac{4}{100} =$ £1·04.

∴ the amount = £50 × $1{\cdot}04^5$ = 50 × 1·21665 = £60 16s. 7¾d.

And the compound interest = £10 16s. 7¾d.

These powers are best calculated by logarithms. See Art. 78.

Thus, log. $1{\cdot}04^5 = 5 \times$ log. $1{\cdot}04 = 5 \times {\cdot}017033 = {\cdot}0851665 =$ log. 1·21665, that is, $1{\cdot}04^5 = 1{\cdot}21665$.

Commercial men usually make these calculations from a Table showing the amount of £1 at compound interest for different numbers of years and at different rates per cent. Thus, from this table, we find the amount of £1, at 4 per cent. for 5 years, to be 1·2166; then this multiplied by the principal, £50, gives the amount required.

2. To find the amount when the interest is payable half-yearly.

In this case, there are 10 terms at which interest is payable, and the amount of £1 for a half-year $= 1 + \frac{2}{100} = 1{\cdot}02$;

∴ the amount = £50 × $1{\cdot}02^{10}$ = £60 18s. 11¾d.

3. Find the amount of; £320 for 4 years at 3 per cent.; £40 for 3 years at 5 per cent.; £1 for 6 years at 3 per cent.

Ans. £360 3s. 3d.; £46 6s. 1d.; £1·194.

4. Find the amount of £20 for 6 years, payable half-yearly, at 5 per cent. per annum. *Ans.* £26 17s. 11¼d.

Rule II.—To find the principal (or present worth) which, at a given rate and for a given time, will amount to a given sum. Divide the given sum by the amount of £1 for the given time and at the given rate.

5. What principal put to interest for 5 years, at 5 per cent. per annum, will amount to £640·262.

Here the amount of £1 for a year is £1·05; then by the 1st rule,

the amount of £1 for 5 years, &c. $= (1{\cdot}05)^5 =$ £1·27628;

∴ the present worth of £1·27628 is £1;

∴ principal, or present worth = £640·262 ÷ £1·27628 = £501 13s. 3d.

6. What principal will amount to £670 1s., at compound interest for 6 years, at 5 per cent. per annum? *Ans.* £500.

7. What principal will amount to £6325 12s., at compound interest for 6 years at 4 per cent. per annum? *Ans.* £5000.

8. A person is to receive the sum of £300 six years hence; what will

be its present worth, allowing 5 per cent. per annum compound interest on the sum paid? *Ans.* £223 17s. $3\frac{1}{4}$d.

9. A person gives £100 for a pipe of wine, containing 50 dozen bottles, what does it cost per dozen if he has to keep it 10 years allowing 5 per cent. per annum compound interest?

Here the amount of £1 for 1 year = 1·05; ∴ the cost of the wine = £100 × $1{\cdot}05^{10}$ = £100 × 1·6289 = £162·89.

∴ Cost per dozen = £162·89 ÷ 50 = £3 5s. $1\frac{3}{4}$d.

Rule III.—To find the number of years which a given principal, at a given rate, will amount to a given sum. Divide the difference of the logarithms of the amount and principal by the logarithm of the amount of £1 for one year.

10. In how many years will £8000 amount to £9261, at 5 per cent.?

Here the amount of £1 for one year = 1·05; and putting n for the number of years required, we have by Rule I.,

$8000 \times 1{\cdot}05^n = 9261$; taking the log. of each side,

log. 8000 + n × log. 1·05 = log. 9261;

∴ n = (log. 9261 — log. 8000) ÷ log. 1·05; which proves the rule,
= (3·9666579 — 3·90309) ÷ ·0211893 = 3 years.

11. In how many years will £250 amount to £270 8s., at 4 per cent. per annum, compound interest? *Ans.* 2.

ANNUITIES.

82. An annuity is a fixed income payable at equal intervals, such as yearly, half-yearly, &c. When an annuity does not commence till a given time has elapsed, it is said to be an *annuity in reversion.* When the annuity is not limited to time, such as the income derived from freehold property, it is called a *perpetuity.*

Rule I.—To find the amount of an annuity, the payments of which are forborn for a given time. Find the amount of one payment at compound interest for the whole time, the interest being chargeable for the interval between the payments; from this subtract the given payment, and divide the remainder by the interest of £1 for the interval between the payments. The general formula is, $\text{A} = \{\text{P}(1 + \text{R})^n - \text{P}\} \div \text{R}$.

To find the amount of an annuity of £8, payable yearly, for 4 years, at 4 per cent. per annum, compound interest.

Here the amount of £1 for 1 year is 1·04. The 1st payment will be in hand 3 years, the 2nd 2 years, the 3rd 1 year, and the 4th will not be chargeable with interest. Hence we have, by the rule of compound interest,

Amount of the 1st payment = £8 × $1{\cdot}04^3$
„ 2nd „ = £8 × $1{\cdot}04^2$
„ 3rd „ = £8 × 1·04
„ 4th „ = £8

∴ The total amount = 8 (1 + 1·04 + $1{\cdot}04^2$ + $1{\cdot}04^3$); by Art. 76,
= 8 ($1{\cdot}04^4$ — 1) ÷ ·04 = (8 × $1{\cdot}04^4$ — 8) ÷ ·04
= £33 19s. $5\frac{1}{4}$d.

Here ·04 is the interest of £1 for one year, and $8 \times 1{\cdot}04^4$ is the amount of one payment at compound interest for 4 years; hence the rule.

If the annuity is payable half-yearly, then there will be 8 payments, and the amount of £1 for a half-year will be £1·02. The 1st payment will be in hand 7 half-years, the 2nd 6 half-years, and so on; hence we find, as before,

$$\begin{aligned}\text{The total amount} &= 8\,(1 + 1{\cdot}02 + \ldots + 1{\cdot}02^7)\\ &= (8 \times 1{\cdot}02^8 - 8) \div {\cdot}02\\ &= £68\ 13s.\ 2d.\end{aligned}$$

Here ·02 is the interest of £1 for a half-year, the interval between the payments; and $8 \times 1{\cdot}02^8$ is the amount of one payment at compound interest for 8 half-years; hence the general rule.

1. What will be the amount of an annuity of £200, payable yearly, for 6 years, at 5 per cent. per annum compound interest?

Here the number of payments is 6, the interval between the payments is 1 year, and the interest of £1 for one year is ·05,

$\therefore$ amnt. of £200 at comp. int. for 6 y. $=$ £200 $\times 1{\cdot}05^6 =$ £268·0192,

$\therefore$ amount $= (268{\cdot}0192 - 200) \div {\cdot}05 =$ £1360 7s. 8d.

2. What will be the amount of an annuity of £9, payable yearly, for 5 years, at 5 per cent, per annum? *Ans.* £49 14s. 7d.

3. What will be the amount of an annuity of £10, payable half-yearly, for 4 years at 4 per cent. per annum?

Here each payment is £10; the number of payments is 8, the interval between the payments being a half-year; and the interest of £1 for a half-year is ·02;

$\therefore$ amount of £10 at comp. int. for 8 y. $=$ £10 $\times 1{\cdot}02^8 =$ £11·7166;

$\therefore$ amount $=$ £$(11{\cdot}7166 - 10) \div {\cdot}02 =$ £85 16s. 7d.

4. What will be the amount of an annuity of £2 10s., payable quarterly, for 4 years, at 4 per cent. per annum? *Ans.* £43 2s. 10½d.

5. A working man, from the age of 21 to 51, had expended £8 a year in beer; how much would he have saved, had he, year after year, put this sum out at 5 per cent. per annum compound interest?

Ans. £531 10s. 2½d.

6. Required the same as in the last example, supposing he had expended £3 per annum in tobacco. *Ans.* £199 6s. 3¾d.

Rule II. To find the present worth of an annuity at compound interest. Find, by the last rule, the amount of the annuity for the given time, and divide this by the amount of £1 at compound interest for the given time.

7. Find the present worth of an annuity of £100 per annum, to be continued 7 years, interest at 4 per cent.

By Rule I., amount annuity $= 100\,(1{\cdot}04^7 - 1) \div {\cdot}04 =$ £789·8294

Amount of £1 at comp. int. $= 1{\cdot}04^7 =$ £1·315932

that is, the present value of £1·315932 $=$ £1

$\therefore$,, ,, £789·8294 $=$ £789·8294 $\div$ 1·315932

$=$ £600 4s. 1¼d.

In order to facilitate these calculations in business, Tables have been published giving the present value of an annuity of £1, from which, the present value of any other annuity is found by multiplication.

8. Find the present value of an annuity of £20 per annum, to be continued 4 years, at 5 per cent. *Ans.* £70 18s. $4\frac{1}{2}$d.

9. Find the present value of a property held on a lease of which 20 years are unexpired, and bringing an annual rent profit of £30, payable yearly, compound interest being allowed at 6 per cent. per annum. *Ans.* £344 1s. $11\frac{1}{4}$d.

10. Required the same as in the last example, supposing only 10 years of the lease unexpired. *Ans.* £220 16s.

11. Find the value of a perpetuity of £30 a year, at 6 per cent. per annum.

Here, the principal to give £6 per an. = £100
∴ „ „ £30 „ = £500.

12. Find the value of a freehold property yielding a profit rental of £63 a year, at 7 per cent. per annum. *Ans.* £900.

Rule III. To find the present value of an annuity in reversion. Find, by Rule II., the present value of the annuity from the present time till the end of its continuance; also find its present value for the time before it comes into possession; and the difference of these will be the present value required.

13. What is the present value of an annuity of £20 a year, to commence 4 years hence, and to continue for the six years succeeding, interest at 4 per cent. per annum?

By Rule II., we find the present value of an annuity of £1 for 10 years at 4 per cent. to be £8·110895, and for 4 years to be £3·63 nearly; therefore the present value required = £(8·110895 — 3·63) × 20 = £89 12s. $4\frac{1}{4}$d.

14. What is the present value of an annuity of £100 a year, to commence 10 years hence, and to continue for the 10 years succeeding, interest at 4 per cent. per annum? *Ans.* £474 0s. $11\frac{1}{4}$d.

15. Required the same as in Ex. 13, allowing the interest to be at 5 per cent. per annum. *Ans.* £83 10s. $3\frac{3}{4}$d.

16. What is the present value of the reversion of a freehold property of £60 per annum, payable yearly, but which does not come into possession till 8 years hence, compound interest being allowed at 6 per cent. per annum?

Present value of the property = $\frac{1}{6}$ of 60 × 100 = £1000.

By Rule II., we find the present value of an annuity of £60 for 8 years at 6 per cent. to be £6·2098 × 60, or £372·588; ∴ the value of the reversion = £1000 — £372·588 = £627 8s. 3d.

17. What would be the value of the reversion when the property is £30 per annum? *Ans.* £313 14s. $1\frac{1}{2}$d.

18. What would be the value of the reversion, in Ex. 16, when the interest is taken at 5 per cent. per annum? *Ans.* £812 4s. $1\frac{3}{4}$d.

19. What fine must be paid to change into freehold, or into perpetuity, a lease for 10 years, which has a profit rental of £40 per annum, payable yearly, compound interest being allowed at 5 per cent. per annum? *Ans.* £491 2s. $7\frac{1}{4}$d.

Here the fine must be equal to the difference of the value of the property, as a freehold, and the value of an annuity of £40 continued for 10 years.

20. What would be the fine, when the interest is taken at 4 per cent. per annum? *Ans.* £675 11s. $3\frac{1}{4}$d.

CONTINUED FRACTIONS.

83. Fractions, even when reduced to their least terms, are often expressed in numbers inconveniently large; but approximate values of them, in smaller numbers, may always be found by the method of continued fractions. An approximate value of a given fraction may be readily found as follows: Thus, $\frac{7}{17} = \frac{1}{2\frac{3}{7}} = \frac{1}{2\frac{1}{2}}$ nearly $= \frac{2}{5}$; again, $\frac{35}{78} = \frac{1}{2\frac{8}{35}} = \frac{1}{2\frac{1}{4}}$ nearly $= \frac{4}{9}$. In like manner to find an approximate value of 3·14; we have, $3{\cdot}14 = 3\frac{14}{100} = 3\frac{7}{50}$; then $\frac{7}{50} = \frac{1}{7\frac{1}{7}} = \frac{1}{7}$ nearly: therefore $3{\cdot}14 = 3\frac{1}{7}$ nearly $= \frac{22}{7}$.

Exercises. Find approximate values of the fractions, $\frac{37}{118}$, $\frac{1096}{1278}$, $\frac{37}{92}$, $\frac{2470}{13581}$.
Answers. $\frac{5}{16}$, $\frac{6}{7}$, $\frac{2}{5}$, $\frac{2}{11}$.

Let it now be required to reduce the fraction $\frac{75}{242}$ to a continued fraction, and thereby to obtain *a series* of approximate values of the fraction each of which approaches more nearly than the one which precedes it to the value of the given fraction.

$\frac{75}{242} = \frac{1}{3\,\frac{17}{75}} = \frac{1}{3\,\frac{1}{4}}$ nearly $= \frac{4}{13}$, which is the 2nd approximate value, the 1st value being $\frac{1}{3}$ found by rejecting $\frac{17}{75}$; here the $\frac{1}{4}$ is found by dividing 75 by 17 and rejecting the remainder. The 3rd approximate value is found by substituting a closer value for $\frac{17}{75}$; thus $\frac{17}{75} = \frac{1}{4\,\frac{7}{17}} = \frac{1}{4\,\frac{1}{2}}$ nearly $= \frac{2}{9}$; $\therefore \frac{75}{242}$ or $\frac{1}{3\,\frac{17}{75}} = \frac{1}{3\,\frac{2}{9}} = \frac{9}{29}$. The 4th value is found by substituting a closer value for $\frac{7}{17}$; thus $\frac{7}{17} = \frac{1}{2\,\frac{3}{7}} = \frac{1}{2\,\frac{1}{2}}$ nearly $= \frac{2}{5}$; $\therefore \frac{17}{75}$ or $\frac{1}{4\,\frac{7}{17}} = \frac{1}{4\,\frac{2}{5}} = \frac{5}{22}$; $\therefore \frac{75}{242}$ or $\frac{1}{3\,\frac{17}{75}} = \frac{1}{3\,\frac{5}{22}} = \frac{22}{71}$. And so on to other values. The values thus found are $\frac{1}{3}$, $\frac{4}{13}$, $\frac{9}{29}$, $\frac{22}{71}$, which approach nearer and nearer to the value of the proposed fraction, also the 1st is greater than the proposed fraction, the 2nd less, the 3rd greater, and so on alternately greater and less, so that the true value lies between any two of these approximate values. These operations conducted according to the usual form of continued fractions, are as follows:

$$\frac{75}{242} = \frac{1}{3 + \frac{17}{75}} = \frac{1}{3 + \cfrac{1}{4 + \frac{7}{17}}} = \frac{1}{3 + \cfrac{1}{4 + \cfrac{1}{2 + \frac{3}{7}}}} = \frac{1}{3 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{2 + \frac{1}{7}}}}}$$

Here rejecting the fraction $\frac{17}{75}$ in the first expression, we find, the 1st approximate value $= \frac{1}{3}$; rejecting the fraction $\frac{7}{17}$ in the second expression, the 2nd approximate value $= \frac{1}{3\,\frac{1}{4}} = \frac{4}{13}$; rejecting the fraction

$\frac{2}{7}$ in the third expression, the 3rd approximate value $= \dfrac{1}{3 + \dfrac{1}{4\frac{1}{2}}} = \dfrac{1}{3\frac{2}{9}}$ $= \frac{9}{29}$; and so on.

The partial denominators, 3, 4, 2, 2, 7, are the quotients obtained by using the rule given, Art. 15, for finding the greatest common measure, as shown in the margin.

```
75)242(3
   225
   ---
    17)75(4
       68
       --
        7)17(2
            &c.
```

Having found the partial quotients, 3, 4, 2, 2, 7, and also the first and second values, $\frac{1}{3}$ and $\frac{4}{13}$, the succeeding values may be readily found in the following manner. Multiply the numerator which occupies one place before that sought by the partial quotient for the latter, and to this product add the numerator two places before; and similarly to find the denominator, multiply the denominator of the place before by the same partial quotient, and to the product add the denominator two places before. Thus, the 1st and 2nd values being $\frac{1}{3}$ and $\frac{4}{13}$, and the 3rd partial quotient 2, the 3rd value $= \dfrac{4 \times 2 + 1}{13 \times 2 + 3} = \frac{9}{29}$; the 4th value $= \dfrac{9 \times 2 + 4}{29 \times 2 + 13} = \frac{22}{71}$; and so on.

As a further application of this rule, let it be required to find a series of approximate values of the fraction $\frac{67}{231}$.

Here the 1st value $= \frac{1}{3}$; the 2nd value $= \dfrac{1}{3\frac{30}{67}} = \dfrac{1}{3\frac{1}{2}}$ nearly $= \frac{2}{7}$; and by Art. 15, we find the partial quotients to be 3, 2, 4, 3, 2; then, the 3rd value $= \dfrac{2 \times 4 + 1}{7 \times 4 + 3} = \frac{9}{31}$; the 4th value $= \dfrac{9 \times 3 + 2}{31 \times 3 + 7} = \frac{29}{100}$; and the 5th value $= \dfrac{29 \times 2 + 9}{100 \times 2 + 31} = \frac{67}{231}$, which is the same as the given fraction.

Exercises. Find a series of approximate values of the fractions; $\frac{52}{276}$; $\frac{43}{223}$; $\frac{419}{896}$; $\frac{1243}{6715}$; $2\frac{67}{363}$; $\frac{37}{100}$.

Answers. $\frac{1}{5}, \frac{3}{16}, \frac{7}{37}$; $\frac{1}{5}, \frac{4}{21}, \frac{13}{68}$; $\frac{1}{2}, \frac{7}{15}, \frac{29}{62}, \frac{65}{139}$; $\frac{1}{5}, \frac{2}{11}, \frac{5}{27}$, &c.; $2\frac{1}{5}, 2\frac{2}{11}, 2\frac{9}{49}$, &c.; $\frac{1}{2}, \frac{1}{3}, \frac{3}{8}$, &c.

OPERATIONS WITH CIRCULATING DECIMALS, ETC.

84. A repeating decimal may be converted into a fraction, and conversely, by the following method:

$$\text{Let } s = \cdot 2\dot{4}\dot{5}45\ldots, \text{ multiplying by } 10,$$
$$10\,s = 2\cdot 4545\ldots, \text{ multiplying by } 100,$$
$$1000\,s = 245\cdot 45\ldots, \text{ subtracting,}$$
$$990\,s = 245 - 2, \therefore s = \frac{245 - 2}{990} = \frac{243}{990}.$$

Hence the rule given in Art. 32. And conversely

$$\tfrac{243}{990} = \tfrac{1}{10}\left(\tfrac{200}{99} + \tfrac{43}{99}\right) = \tfrac{1}{10}\left(2 + \frac{43 + 2}{99}\right) = \cdot 2\dot{4}\dot{5}.$$

Similarly, $\tfrac{237}{99} = \tfrac{200}{99} + \tfrac{37}{99} = 2 + \dfrac{37 + 2}{99} = 2\cdot\dot{3}\dot{9}.$

This converse rule fails when the addition changes any of the figures in the part which does not circulate; in this case we may proceed as follows:

$$\frac{5432732}{99900} = \tfrac{1}{100} \times \frac{5432732}{999} = \tfrac{1}{100}\left(5438 + \tfrac{170}{999}\right) = 54\cdot 38\dot{1}7\dot{0}.$$

Here the division by 999 is much facilitated by the following *general rule for dividing by a number composed of* 9's.

$$\begin{array}{r|l} 5432 & 732 \div 999 \\ 5 & 432 \\ & \quad 5 + 1 \\ \hline 5438 & 170 \end{array}$$

Here as there are *three* 9's in the divisor, we cut off *three* figures from the dividend; we now set down the quotient figures, thus found, beneath the remainder figures, giving 5 for the next quotient; this second quotient is set down as before; and so on. The columns are added together, and 1 being carried, from the addition of 4 and 7, to the units' column of the quotients, we add 1 to the remainder. The true quotient, therefore, is is 5438 with the remainder 170. As the divisor 999 is equivalent to 1000 — 1, the division of any four figures by 1000 always leaves a remainder equal to the quotient; hence the operation above given.

85. In certain arithmetical problems it is requisite that the value of repeating decimals should be accurately determined. The following methods of calculation, it is believed, will be found interesting as well as useful.

For the sake of convenience, for 9999 we shall write 9..9, where the number of dots indicate the number of 9's omitted.

The repeating figures may be written down any number of times without altering the value of the quantity:

Thus $\cdot\dot{3}\dot{5} = \tfrac{35}{99} \times \tfrac{101}{101} = \dfrac{3535}{9..9} = \cdot\dot{3}53\dot{5}.$

Similarly, $\cdot 2\dot{3}\dot{4} = \cdot 2\dot{3}43\dot{4} =$ &c.

86. To convert circulating decimals to fractions of the same denominator.

To reduce $\cdot\dot{4}2\dot{3}$, $\cdot\dot{3}\dot{1}$, $\cdot\dot{2}$ to fractions of the same denominator.

$$\cdot\dot{4}2\dot{3} = \frac{423423}{9....9},\ \cdot\dot{3}\dot{1} = \frac{313131}{9....9},\ \cdot\dot{2} = \frac{2....2}{9....9}.$$

Here the 1st circulate is written down twice, the 2nd thrice, and the 3rd six times, giving a common denominator of six 9's. The number of 9's in the common denominator will be the least common multiple of the numbers of the repeating figures in the respective parts. In this example, the numbers of the repeating figures, in the parts, are 3, 2, 1, and as the least common multiple of these numbers is 6, there are six 9's in the denominator.

Again, we have for the mixed circulators $\cdot 31\dot{6}8\dot{5}$, $\cdot\dot{2}\dot{7}$.

$$\cdot 31\dot{6}8\dot{5} = \frac{31685685 - 31}{9\ldots\ldots 900} = \frac{31685654}{9\ldots\ldots 900};\ \cdot\dot{2}\dot{7} = \frac{27272700}{9\ldots\ldots 900}.$$

87. To add and subtract circulating decimals.

Let it be required to add $\cdot 3\dot{2}\dot{5}$, $\cdot 23\dot{5}1\dot{3}$, $\cdot\dot{3}$. Here we bring the proposed decimals to fractions having the same denominator as explained in Art. 86. Then adding the numerators together and transforming the fraction into the form of a decimal, Art. 84, we obtain the sum required.

$$\begin{array}{r} 3252522 \\ 2351349 \\ 3333330 \\ \hline \dfrac{8937201}{9\ldots\ldots 90} \\ = \cdot\dot{8}93720\dot{9} \end{array}$$

Again, $\cdot 56\dot{2}\dot{3} - \cdot 2\dot{4}\dot{5} = \dfrac{5567 - 2430}{9900} = \frac{3137}{9900} = \cdot 31\dot{6}\dot{8}$.

88. To multiply circulating decimals.

Let it be required to multiply $\cdot\dot{1}\dot{4}$ by $\cdot\dot{3}0\dot{2}$.

$$\frac{14}{99} \times \frac{302}{999} = \frac{4228}{99\,(1000 - 1)} = \frac{4228}{99000 - 99} = \frac{4228}{98901}.$$

Here the result is given in the form of a vulgar fraction, but it may be readily expressed in the form of a decimal by dividing the product of the numerators first by 99 and then by 999, according to the method given in Art. 84: thus

$$\begin{array}{r|l} 42 & 28 \div 99 \\ & 42 \\ \hline 42 & 70 = 42\cdot\dot{7}\dot{0} \end{array} \qquad\qquad \begin{array}{r|l} 42\cdot 707 & 070 \div 999 \\ 42 & 707 \\ & 42 \\ \hline \cdot 042749 & 819 \end{array}$$

Here the quotient of the first division is $42\frac{70}{99}$ or $42\cdot\dot{7}\dot{0}$. In the second operation this quotient is written down as the dividend with the repeating figures extended at pleasure, and the number of decimal figures in the quotient is the same as the number employed in the dividend.

Similarly, $\cdot\dot{3}\dot{2} \times 6141 = \frac{196512}{99} = 1984\cdot\dot{9}\dot{6}$.

89. To divide circulating decimals.

Let it be required to divide $\cdot 6\dot{1}\dot{7}$ by $\cdot\dot{2}1\dot{4}$. Here by Art. 86, we first reduce the decimals to fractions having the same denominator:

$$\cdot 6\dot{1}\dot{7} = \cdot 6\dot{1}717\dot{1}\dot{7} = \frac{6171711}{9\ldots\ldots 90};\ \cdot\dot{2}1\dot{4} = \frac{2142140}{9\ldots\ldots 90};$$

$$\therefore\ \cdot 6\dot{1}\dot{7} \div \cdot\dot{2}1\dot{4} = \frac{6171711}{9\ldots\ldots 90} \div \frac{2142140}{9\ldots\ldots 90} = \frac{6171711}{2142140}.$$

Similarly, $\cdot 3\dot{4}\dot{5} \div \cdot 33\dot{6} = \frac{3420}{9900} \div \frac{3333}{9900} = 1\cdot\dot{0}26\dot{1}$.

Again, $28\cdot\dot{4}\dot{3} \div 3\cdot\dot{2}\dot{7} = \frac{2815}{99} \div \frac{324}{99} = \frac{2815}{324}$.

Properties of Divisors and Factors.

90. Any even number is divisible by 2. A number is divisible by 3 when the sum of its digits are so (see the properties of 9). A number is divisible by 4, when the two last figures are so: thus $1356 = 13 \times 100 + 56$; where the first part is divisible by 4, because 100 is so; but the second part being divisible by 4, therefore the whole is so. A number is divisible by 8 when the three last figures are so: thus $7296 = 7 \times 1000 + 296$; where 1000 is divisible by 8, and so on as before. A number is not divisible by 5, unless its last figure is 0 or 5. A number is divisible by 6, when it is divisible by 3 and when it is even.

91. If a number be divided into periods of three figures each, beginning at the units' place, then the number will be divisible by 7 when the sums of the alternate periods are equal, or their difference is a multiple of 7. Thus 337,568,231 is divisible by 7, where the sum of the 1st and 3rd periods is equal to the 2nd period, as shown in the following proof:

$$337568231 = 337337000 + 231231 = 337 \times 1001000 + 231 \times 1001.$$

Here the proposed number is decomposed into two parts, where 1001, which is a factor of the two parts, is divisible by 7: therefore the sum, making up the proposed number, is also divisible by 7. Moreover any multiple of 7 added to or subtracted from any of these periods will produce a number also divisible by 7.

92. A number is divisible by 9, when the sum of its digits is so; thus 378 is divisible by 9, where the sum of the digits is 18 or 2×9. Moreover, dividing any number by 9 will leave the same remainder as dividing the sum of its digits by 9; thus the remainder from dividing 534 by 9 is three, also the sum of the digits, 12, divided by 9 leaves the remainder 3. These properties may be proved in the following manner:

$$\begin{aligned} 500 &= 5 \times (99 + 1) = 5 \times 99 + 5 \\ 30 &= 3 \times (9 + 1) = 3 \times 9 + 3 \\ 4 &= + 4 \\ \hline 534 &= 5 \times 99 + 3 \times 9 + 5 + 3 + 4 \end{aligned}$$

$$\therefore 534 \div 9 = 5 \times 11 + 3 + (5 + 3 + 4) \div 9,$$

where the remainder from the division of the number is equal to the remainder from the division of the sum of the digits. When the sum of the digits is a multiple of 9 there is no remainder and then the proposed number is divisible by 9. Hence we have the following properties:

When two numbers are expressed by the same figures, but differently arranged, then their difference will be divisible by 9; thus $523 - 325 = 198$. Similarly, if from any number the sum of its digits be subtracted, the remainder will be divisible by 9; thus $357 - 15 = 342$. Also, if the number composed of the nine digits be multiplied by 9, or any multiple of 9 not exceeding 9×9, then the product will be composed of the same figures except the figure in the tens' place which will be 0; thus $123 \ldots . 89 \times 9 \times 4 = 44 \ldots 404$.

93. If the sum of the digits in the odd places be equal to the sum of the digits in the even places, or differ by 11 or a multiple of 11, then

the number is divisible by 11. Thus 4312 is divisible by 11, where $2+3 = 5$, and $1 + 4 = 5$; also 90827, where $7 + 8 + 9 = 2 \times 11 + 2$, and $2 + 0 = 2$. Proof. In this example, the sum of the odd digits in the product $= 5 + 2 + 4 + 3$, and the sum of the even digits $= 5 + 2 + 4 + 3$; but these sums are the same. Again, in the following example, the digits in the product are 7, $6 + 7 - 10, 2 + 6 + 1, 2$; but the sum of the odd digits $= 7 + 2 + 6 + 1$, and the sum of the even digits $= 6 + 7 - 10 + 2$; adding 10 to each, these sums respectively become $2 + 6 + 7 + 11$, and $2 + 6 + 7$, where the former exceeds the latter by 11.

$$\begin{array}{r} 3425 \times 11 \\ 3425 \\ \hline 37675 \end{array} \qquad \begin{array}{r} 267 \times 11 \\ 267 \\ \hline 2937 \end{array}$$

94. The following *abbreviated forms of calculation* are worthy of notice:

To multiply by 5: *add a cipher to the multiplicand and divide by* 2. To multiply by 15: *add a cipher to the multiplicand, and to the result add half of itself.* To multiply by 25: *add two ciphers and divide by* 4. To multiply by 75: *add two ciphers, and from this result take one-fourth of itself.* And so on to other factors which are simple fractions of 100, 1000, &c.

To multiply by 9: *add a cipher to the multiplicand, and subtract the multiplicand.* To multiply by a number composed of 9's: *add as many ciphers to the multiplicand as there are 9's in the multiplier and from this result subtract the multiplicand:* thus $346 \times 99 = 346 \times (100 - 1) = 34600 - 346 = 34254$. To multiply by a number such as 698, which wants 2 to make it 700: *multiply by* 700 *and* (in this case) *subtract* 2 *times the multiplicand.* To multiply by 49, or 499, or 4999, . . . : *add as many ciphers to the multiplicand as there are figures in the multiplier, divide by* 2, *and subtract the multiplicand.* To multiply by a number such as 594, where there is a central 9 and the other figures make up 9: *add* 1 *to the figure in the hundreds' place and multiply by it, add two ciphers to the product and subtract the product itself.*

To divide by 5: *double the dividend and cut off one decimal figure.* To divide by 25: *multiply the dividend by* 4 *and cut off two decimal figures.* To divide by 125: *multiply the dividend by* 8 *and cut off three decimal figures.* To divide by 75: *multiply the dividend by* 4 *and divide the product by* 300. And so on to other cases, where the divisor is some simple fractional part of 100, 1000, &c.

To divide by a number a little less than 100, 1000, &c. The rule, in this case, is a general form of that given in Art. 84.

Here the divisor 98 wants 2 to make up 100. We add ciphers at pleasure to the dividend. In the place of adding the quotients to the remainders, as in Art. 84, we multiply these quotients by 2, and at last add 2 times the number carried from the last column of the remainders. As there are four decimal figures in the dividend we mark off the same number in the quotient, so that the result of the division is 1·4933 with the remainder 66.

$$\begin{array}{r|l} 146{\cdot}35 & 00 \div 98 \\ 292 & 70 \\ 5 & 84 \\ & 10+2 \\ \hline 1{\cdot}4933 & 66 \end{array}$$

Proof. By Art. 80, the sum of the geometric series,

$\frac{a}{100} + \frac{a}{100} \times \frac{2}{100} + \frac{a}{100} \times \frac{2^2}{100^2} +$ &c. $= \frac{a}{98}$, where any term of the series is found from the term preceding it by multiplying by 2 and dividing by 100.

To divide by a number composed of 9's with any other figure in the last place, such as 299: *Increase the last figure by* 1, *and add as many ciphers to it as there are 9's in the divisor; successively divide by the number thus formed and add the results together.*

Here the last figure in the divisor is 2, which increased by 1 is 3, therefore, we successively divide by 300, and add the quotients. The operation, in this case, may be carried on without end. As in the foregoing proof, we find, $\frac{a}{300} + \frac{a}{300^2} + \frac{a}{300^3} +$ &c. $= \frac{a}{299}$.

$$
\begin{array}{l}
84240 \div 299 \\
\hline
280{\cdot}80 \\
\quad {\cdot}936 \\
\quad {\cdot}00312 \text{ \&c.} \\
\hline
281{\cdot}7391
\end{array}
$$

95. The following properties prove the rules of construction, given in Art. 77 of the Author's "Principles of Arithmetic."

$$357 \times 999 = 357\,(1000 - 1) = 357000 - 357 = 356643,$$

where the three left hand figures in the product are obtained by subtracting 1 from the multiplicand, and the other figures are supplemental, that is, make up nines in the usual manner. Conversely

$$3762 = 3700 + 99 - 37 = 37 \times 99 + 99 = 38 \times 99$$

Again, $75 \times 999 = 75\,(1000 - 1) = 75000 - 75 = 74925$, where there is one central 9 with the usual law of nines. Conversely $42957 = 43000 - 43 = 43 \times 999$. And so on.

$$13{,}86 + 25{,}74 = 14 \times 99 + 26 \times 99 = 40 \times 99,$$

that is, the sum follows the usual law of nines. Again, to subtract 1265 from 3245, where the figures make up sevens. Adding and subtracting 22, we have, $3245 - 1265 = 3245 + 22 - 1265 - 22 = 3267 - 1287 = 33 \times 99 - 13 \times 99 = 20 \times 99$, that is, the difference follows the usual law.

$$2475 \times 3 = 25 \times 99 \times 3 = 75 \times 99, \text{ also}$$
$$25974 \times 34 = 26 \times 999 \times 34 = 884 \times 999$$

where the products follow the usual law of nines.

$$13875 \times 9 = 138750 - 13875 = 124875,$$

where the multiplier is 9, and the multiplicand has a central 8, the other figures make up eights in the usual manner, and the product follows the law of nines. Hence the product of 13875 by 36 will also follow the same law.

LONDON:
F. SHOBERL, PRINTER, 37, DEAN STREET, SOHO, W.